Math 5/4, Math 6/5, Math 7/6, Math 8/7, and Algebra ½ form a series of courses to move students from primary grades to algebra. Each course contains a series of daily lessons covering all areas of general math. Each lesson presents a small portion of math content (called an increment) that builds on prior knowledge and understanding.

Students are not required or expected to grasp a concept fully the first time it is presented. After an increment is introduced, it becomes a part of the student’s daily work for the rest of the year. Students will have many opportunities to gain understanding and to achieve mastery. This cumulative, continual practice ensures that students will retain what they have learned.

This sampler includes materials that are representative of the Saxon math program, including samples of Lessons and Investigations.

We hope these materials will assist you in your evaluation of the Saxon program.
# Table of Contents

*Math 5/4,* .......................................................... 3  
  Lesson 40, Capacity ........................................ 4  
  Lesson 57, Rate Word Problems ............................. 9  
  Lesson 83, Sales Tax • Change Back ......................... 13  
  Investigation 8, Graphing Relationships ................... 17  

*Math 6/5,* .......................................................... 20  
  Lesson 53, Perimeter • Measures of a Circle ............... 21  
  Lesson 80, Prime and Composite Numbers ................ 26  
  Lesson 91, Simplifying Improper Fractions ................ 31  
  Investigation 11, Scale Drawings ............................ 36  

*Math 7/6,* .......................................................... 39  
  Lesson 62, Writing Mixed Numbers as Improper Fractions 40  
  Lesson 89, Estimating Square Roots ......................... 46  
  Lesson 104, Algebraic Addition Activity ................... 52  
  Investigation 7, The Coordinate Plane ....................... 58  

*Math 8/7,* .......................................................... 63  
  Lesson 35, Adding, Subtracting, Multiplying, and Dividing Decimal Numbers ........................................ 64  
  Lesson 65, Ratio Problems Involving Totals ................ 71  
  Lesson 107, Slope ........................................... 77  
  Investigation 5, Creating Graphs ............................. 86  

*Algebra ½,* ....................................................... 90  
  Lesson 59, Proportions with Fractions ...................... 91  
  Lesson 75, Implied Ratios .................................. 94  
  Lesson 105, Evaluating Powers of Negative Bases .......... 98
Math 5/4
Table of Contents

Lesson 40, Capacity ......................................................... 4
Lesson 57, Rate Word Problems ......................................... 9
Lesson 83, Sales Tax • Change Back ................................. 13
Investigation 8, Graphing Relationships ............................. 17
LESSON 40
Capacity

WARM-UP

Facts Practice: Multiplication Facts: 2’s, 5’s, Squares (Test D)
Mental Math:
Practice adding 99c, 96c, or 95c to money amounts:
a. $5.85 + $0.99  b. $8.63 + $0.98  c. $4.98 + $0.95
Review:
d. 574 – 200  e. 77 + 6 + 110  f. 460 + 300 + 24
Problem Solving:
The two hands of a clock are together at noon. The next time the hands of a clock are together is about how many minutes later?

NEW CONCEPT

Liquids such as milk, juice, paint, and gasoline are measured in the U.S. Customary System in fluid ounces, cups, pints, quarts, or gallons. This table shows the abbreviations for each of these units:

<table>
<thead>
<tr>
<th>Abbreviations for U.S. Liquid Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>fluid ounce</td>
</tr>
<tr>
<td>cup</td>
</tr>
<tr>
<td>pint</td>
</tr>
<tr>
<td>quart</td>
</tr>
<tr>
<td>gallon</td>
</tr>
</tbody>
</table>

The quantity of liquid a container can hold is the capacity of the container.

Activity: Measuring Capacity

Materials needed:
- empty, clean plastic or paper containers of the following sizes (with labels that show the container’s size): 1 gallon, 1 half gallon, 1 quart, 1 pint, 1 cup, and 1 liter (or 2 liters)
- supply of water
- funnel
Place five liquid containers (gallon, half gallon, quart, pint, and cup) on a table. Arrange the containers in order from smallest to largest.

1 cup 1 pint 1 quart 1/2 gallon 1 gallon

Estimate the number of cups of liquid needed to fill the 1-pint container. Then fill the 1-pint container with water using the 1-cup container. Was your estimate correct?

Next, estimate the number of pints needed to fill the 1-quart container. Check your estimate by filling the 1-quart container with water using the 1-pint container. Continue the process through the largest container (gallon), and use your findings to answer the following questions:

a. How many cups of liquid equal a pint?

b. How many pints of liquid equal a quart?

c. How many quarts of liquid equal a half gallon?

d. How many half gallons of liquid equal a gallon?

e. How many quarters equal a dollar?

f. How many quarts of liquid equal a gallon?

g. Copy and complete this table of U.S. Customary liquid measures. Notice that 8 fluid ounces equals 1 cup.

<table>
<thead>
<tr>
<th>U.S. LIQUID MEASURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 fl oz = 1 c</td>
</tr>
<tr>
<td>1 c = 1 pt</td>
</tr>
<tr>
<td>1 pt = 1 qt</td>
</tr>
<tr>
<td>1 qt = 1 gal</td>
</tr>
</tbody>
</table>

Liquids are also measured in liters (abbreviation, L). A liter is a metric unit of measure. Compare a one-liter container to a one-quart container (or compare a two-liter container to a half-gallon container). Which container looks larger?
Use a full liter (or two-liter) container to fill a quart (or half-gallon) container. Then complete these comparisons:

h. Compare: 1 quart $\bigcirc$ 1 liter

i. Compare: $\frac{1}{2}$ gallon $\bigcirc$ 2 liters

To measure small amounts of liquid, we may use **milliliters** (mL). Droppers used for liquid medicine usually hold one or two milliliters of liquid. One thousand milliliters equals one liter.

**METRIC LIQUID MEASURE**

\[
1000 \text{ mL} = 1 \text{ L}
\]

j. A full 2-liter bottle of liquid contains how many milliliters of liquid?

Inspect the labels of the liquid containers used in the activity. Liquid containers often list two measures of the quantity of liquid the containers hold. For example, the label on a one-gallon milk bottle may read

1 gal (3.78 L)

The measure 3.78 L means $3\frac{78}{100}$ liters. The number 3.78 is a decimal number. Decimal numbers are often used in measurement, especially in metric measurement. The number 3.78 has a whole-number part, the 3, and a fraction part, the .78. So 3.78 L means “more than three liters but a little less than four liters,” just as $3.78$ means “more than three dollars but not quite four dollars.” We read 3.78 as “three and seventy-eight hundredths.” We will learn more about decimal numbers in Investigation 4.

**MIXED PRACTICE**

Problem set

1. A group of quail is called a **covey**. A group of cows is called a **herd**. A group of fish is called a **school**. There are twenty-five fish in the small school. There are one hundred twelve fish in the big school. How many fewer fish are in the small school?

2. A 36-inch yardstick was divided into two pieces. One piece was 12 inches long. How many inches long was the other piece?
3. Mrs. Green mailed forty-seven postcards from Paris. Her husband mailed sixty-two postcards from Paris. Her son mailed seventy-five postcards from Paris. In all, how many postcards did the Greens mail from Paris?

4. Write the number 7,500,000 in expanded form. Then use words to write the number.

5. Which digit in 27,384,509 is in the thousands place?

6. Use a dollar sign and a decimal point to write the value of three dollars, two quarters, one dime, and two nickels. Then write that amount of money using words.

7. A gallon of milk is how many quarts of milk?

8. How many squares are shaded?

9. Use a ruler to find the length of the line segment below to the nearest quarter inch.

10. Printed on the label of the milk container were these words and numbers:

   1 gal (3.78 L)

   Use this information to compare the following:

   1 gallon ○ 3 liters

11. It is evening. What time will it be 1 hour and 50 minutes from now?

12. In problem 11 what type of angle is formed by the hands of the clock?

   A. acute    B. right    C. obtuse
13. Compare:
   (a) \(-29 \bigcirc -32\)  
   (b) \(0.75 \bigcirc \frac{3}{4}\) of a dollar

14. Draw a circle with a diameter of 2 centimeters. What is the radius of the circle?

Multiply:
15. (a) \(6 \times 6\)  
   (b) \(7 \times 7\)  
   (c) \(8 \times 8\)

16. (a) \(7 \times 9\)  
   (b) \(6 \times 9\)  
   (c) \(9 \times 9\)

17. (a) \(7 \times 8\)  
   (b) \(6 \times 7\)  
   (c) \(8 \times 4\)

18. \(4.98 + 7.65\)

19. \(M - 6.70 = 3.30\)

20. \(416 - Z = 179\)

21. \(536 + Z = 721\)

22. \(\sqrt{1} + \sqrt{4} + \sqrt{9}\)

23. Draw an array of X's to show \(3 \times 7\).

24. Use words to write \(10\frac{1}{10}\).

25. (a) Two quarters are what fraction of a dollar?
   
   (b) Write the value of two quarters using a dollar sign and a decimal point.

26. A rectangle has an area of 24 square inches. Which of these could be the length and width of the rectangle?
   A. 6 in. by 6 in.  
   B. 12 in. by 12 in.  
   C. 8 in. by 4 in.  
   D. 8 in. by 3 in.

27. Robert measured the width of his notebook paper and said that the paper was \(8\frac{3}{4}\) inches wide. What is another way to write \(8\frac{3}{4}\)?
LESSON 57
Rate Word Problems

WARM-UP

Facts Practice: 90 Division Facts (Test J)
Mental Math:
Subtract dollars and cents from dollars:
a. $1.00 - $0.85  
b. $2.00 - $0.63  
c. $5.00 - $1.25
Review:
d. 340 + 500 + 32  
e. $5.47 + $1.95  
f. 400 - 30
Problem Solving:
In this addition problem some digits are missing. Copy this problem on your paper, and fill in the missing digits.

53 + 28

50

NEW CONCEPT

A rate shows a relationship between two different measurements. Here we relate the measurements “miles” and “hours”:

The car went 30 miles per hour.

This statement tells us that the car’s rate is 30 miles each hour. Each hour can be considered one “time group.” We will see in the following examples that rate problems have the same pattern as “equal groups” problems.

Example 1  Liam drove the car 30 miles per hour for 4 hours. How far did Liam drive?

Solution  This is a rate problem. A rate problem is about “equal groups.”

We do not see the words in each in a rate problem, but there are words that mean in each. The words miles per hour in this problem mean “miles in each hour.”

\[
\begin{array}{c|c}
\text{Pattern} & \text{Problem} \\
\hline
\text{Number in each time group} & 30 \text{ miles per hour} \\
\times \text{Number of time groups} & \times 4 \text{ hours} \\
\text{Total} & 120 \text{ miles} \\
\end{array}
\]

Liam drove 120 miles.
Example 2  Nalcomb earns 3 dollars a week for doing chores. How much money does he earn for doing 7 weeks of chores?

Solution  This is a rate problem. A rate problem is an “equal groups” problem. The phrase 3 dollars a week means “3 dollars each week.”

Pattern:  
Number of groups × number in each group = total

Problem:  
7 weeks × 3 dollars per week = 21 dollars

Nalcomb earns 21 dollars for doing 7 weeks of chores.

LESSON PRACTICE

Practice set  
a. Kali drove 55 miles in one hour. At that rate, how far can she drive in 6 hours? Write a multiplication pattern and solve the problem.

b. Jeff swims 20 laps every day. How many laps will he swim in 1 week? Write a multiplication pattern and solve the problem.

MIXED PRACTICE

Problem set  
1. Marybeth could jump 42 times each minute. At that rate, how many times could she jump in 8 minutes? Use a multiplication pattern to solve the problem.

2. Robo could run 7 miles in 1 hour. At that rate, how many miles could Robo run in 3 hours? Use a multiplication pattern to solve the problem.

3. Write four multiplication/division facts using the numbers 8, 9, and 72.

4. What is the sum of √36 and √64?

5. Compare: \( \frac{1}{3} \bigcirc 50\%

6. (a) Round 5280 to the nearest thousand.  
   (b) Round 5280 to the nearest hundred.
7. It is afternoon. What time was it 6 hours and 5 minutes ago?

8. Find the fourth multiple of 6. Then find the third multiple of 8. What is the sum of these two multiples?

9. How many years were there from 1492 until 1800? Use a subtraction pattern to solve the problem.

10. A square has one side that is 7 inches long.
   (a) What is the perimeter of the square?
   (b) What is the area of the square?

11. $\frac{70,003}{36,418}$ = $\frac{N}{4.32}$ $\frac{13.861.34}{2.57}$ $\frac{13.861.34}{2.57}$

12. $\frac{93}{5}$ $\frac{84}{6}$ $\frac{77}{7}$ $\frac{80}{8}$

13. $N$ $\frac{45}{6}$ $\frac{56}{8}$ $\frac{7}{65}$

21. $7N = 42$

22. $1.75 + 17.5$

23. (a) Which segment in this figure is a diameter?
   (b) Segments MW and MX form an angle. What type of angle is it?

24. Compare these fractions. Draw and shade two congruent rectangles to show the comparison.
25. Point X represents what number on this number line?

26. One inch is 2.54 centimeters. A segment that is 3 inches long is how many centimeters long?

27. Write this addition problem as a multiplication problem:
   \[2.54 + 2.54 + 2.54\]

28. (a) Three pennies are what fraction of a dollar?
    (b) Three pennies are what percent of a dollar?
    (c) Write the value of three pennies as a decimal part of a dollar.
Math 5/4, Lesson 83
Sample taken from Math 5/4 (Third Edition), page 388

LESSON 83
Sales Tax • Change Back

WARM-UP

Facts Practice: 90 Division Facts [Test I]
Mental Math:
Counting by fives from 1, 2, 3, 4, or 5, we find five different final-digit patterns: 1 and 6; 2 and 7; 3 and 8; 4 and 9; and 5 and 0. When a number ending in 5 is added to or subtracted from another number, the final digit of that number and of the answer will fit one of the five patterns. Look for the final-digit patterns in these problems:

\[
\begin{align*}
a. & \quad 22 + 5 \\
b. & \quad 22 - 5 \\
c. & \quad 38 + 5 \\
d. & \quad 38 - 5 \\
e. & \quad 44 + 5 \\
f. & \quad 44 - 5 \\
\end{align*}
\]

NEW CONCEPTS

Sales tax Sales tax is an extra amount of money that sometimes must be paid when items are purchased. The amount of tax depends upon the amount purchased and the local sales-tax rate. In the United States sales-tax rates vary by city, by county, and by state.

Example 1 Yin bought six bolts priced at 89¢ each. The total sales tax was 32¢. How much did Yin spend in all?

Solution First we find the cost of the six bolts by multiplying.

\[
\begin{align*}
& \underline{89\text{¢}} \\
\times & \underline{6} \\
\hline
& \underline{534\text{¢} = \$5.34} \\
\end{align*}
\]

The six bolts cost $5.34. Now we add the sales tax.

\[
\begin{align*}
& \underline{\$5.34 \quad \text{cost of bolts}} \\
+ & \underline{\$0.32 \quad \text{sales tax}} \\
\hline
& \underline{\$5.66 \quad \text{total cost}} \\
\end{align*}
\]

The total cost, including tax, was $5.66.

Example 2 Pam bought a blouse priced at $25. The sales-tax rate was 8¢ per dollar. How much tax did Pam pay?
Solution  The tax was 8¢ for each dollar of the purchase price. So the tax on $1 was 8¢, the tax on $2 was 16¢, the tax on $3 was 24¢, and so on. To find the tax on $25, we multiply 25 by 8¢.

\[ 25 \times 8\text{¢} = 200\text{¢} \]

Since 200¢ is two dollars, Pam paid a tax of $2.00 on the blouse.

Change back  If we do not have the exact amount of money needed to buy something at a store, we pay more than the total cost and then we get change back. To find how much change we should get back, we subtract the total cost from the amount we paid.

Example 3  Midge bought a pair of pants priced at $23.99. The sales tax was $1.56. Midge paid the clerk $40.00. How much money should she get back in change?

Solution  First we figure out the total cost.

\[
\begin{array}{c}
23.99 & \text{price of pants} \\
+ 1.56 & \text{sales tax} \\
--- & --- \\
25.55 & \text{total cost}
\end{array}
\]

Now we subtract the total cost from the amount she paid.

\[
\begin{array}{c}
40.00 & \text{amount paid} \\
- 25.55 & \text{total cost} \\
--- & --- \\
14.45 & \text{change back}
\end{array}
\]

Midge should get $14.45 back from the clerk.

LESSON PRACTICE

Practice set  a. Sarah bought three pairs of socks. Each pair was priced at $2.24. The total sales tax was 34¢. Altogether, how much did Sarah spend on socks?

b. Hakim paid $10.00 for a tape that cost $6.95. The sales tax was 49¢. How much money should Hakim get back in change?

MIXED PRACTICE

Problem set  1. Blackbeard brought home 30 bags. Each bag contained 320 gold coins. How many coins were there in all?

2. The movie was 3 hours long. If it started at 11:10 a.m., at what time did it end?

3. Jeremy is reading a 212-page book. If he has finished page 135, how many pages does he still have to read?
Math 5/4, Lesson 83
Sample taken from Math 5/4 (Third Edition), page 390

4. Brad, Jan, and Jordan each scored one third of the team's 42 points. They each scored how many points?

5. Round 4286 to the nearest thousand.

6. The shirt was priced at $16.98. The tax was $1.02. Sam paid the clerk $20. How much money should Sam get back?

7. What fraction of the letters in the following word are I's?

SUPERCALIFRAGILISTICEXPIALIDOUCIOUS

Use the information below to answer problems 8–10:

In the first 8 games of this season, the Rio Hondo football team won 6 games and lost 2 games. They won their next game by a score of 24 to 20. The team will play 12 games in all.

8. In the first nine games of the season, how many games did Rio Hondo win?

9. Rio Hondo won its ninth game by how many points?

10. What is the greatest number of games Rio Hondo could win this season?

11. Compare: $3 \times 4 \times 5 \bigcirc 5 \times 4 \times 3$

12. $M - 137 = 257$

13. $N + 137 = 257$

14. $1.45 + 2.4 + 0.56 + 7.6$

15. $5.75 - (3.12 + 0.5)$

16. $638 \div 50$

17. $472 \div 9$

18. $\$6.09 \times 6$

19. $3\overline{921}$

20. $5\overline{678}$

21. $4\overline{2400}$
22. $12.60 \div 5$
23. $14.34 \div 6$
24. $46.00 \div 8$

25. $9^2 = 9 \times N$
26. $5 \times W = 5 \times 10^2$

27. The names of one fourth of the months begin with the letter J. What percent of the months begin with the letter J?

28. What is the perimeter of this rectangle in millimeters?

29. Draw a rectangle that is similar to the rectangle in problem 28 and whose sides are twice as long. What is the perimeter in centimeters of the rectangle you drew?

30. Kurt spun around three times and then fell down dizzy. How many degrees did Kurt turn?
**Focus on**

**Graphing Relationships**

Tables and graphs can be used to display relationships between two quantities, such as pay and time worked.

Suppose Dina has a job that pays $10 per hour. This table shows the total pay Dina would receive for 1, 2, 3, or 4 hours of work.

1. Copy the table. Extend the table to show Dina's pay for each hour up to 8 hours of work.

<table>
<thead>
<tr>
<th>Hours Worked</th>
<th>Total Pay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10</td>
</tr>
<tr>
<td>2</td>
<td>$20</td>
</tr>
<tr>
<td>3</td>
<td>$30</td>
</tr>
<tr>
<td>4</td>
<td>$40</td>
</tr>
</tbody>
</table>

The graph below shows the same relationship between hours worked and total pay. Each dot on the graph represents both a number of hours and an amount of pay.

2. Copy the graph. Extend the sides of the graph to include 8 hours and $80. Then graph (draw) the dots for Dina's total pay for each hour up to 8 hours.

Professor Smith writes the percent of correct answers on each test and quiz she grades. These tables show percent scores for 10-question quizzes and 20-question tests:
Math 5/4, Investigation 8
Sample taken from Math 5/4 (Third Edition), page 377

3. Copy the table for 20-question tests. Extend the table to show scores for each number of correct answers up to 20.

This graph shows the relationship between correct answers and percent scores for a 20-question test. Refer to the graph to answer the questions that follow.

4. Sonia answered 18 questions correctly. What was her percent score?

5. Frank scored 75%. How many correct answers did Frank have?
Sometimes we want to name points on a grid. Below we show how to name points using pairs of numbers called coordinates. The first number in each coordinate pair is taken from the horizontal scale. The second number in each pair is taken from the vertical scale. We write the coordinates in parentheses.

6. Write the coordinates of point A.

7. Write the coordinates of point B.

To draw this star, we connect points by using segments. We start at point A, draw a segment to point B, and then continue in order to points C, D, and E before going back to point A.

**Activity: Graphing on a Coordinate Grid**

Materials needed:

- Activity Sheet 26 (available in the *Saxon Math 5/4—Homeschool Tests and Worksheets*)

Practice graphing points on a grid and connecting the points to complete a design.
Math 6/5

Table of Contents

Lesson 53, Perimeter • Measures of a Circle ......................... 21
Lesson 80, Prime and Composite Numbers ......................... 26
Lesson 91, Simplifying Improper Fractions ......................... 31
Investigation 11, Focus On Scale Drawings ......................... 36
LESSON 53  Perimeter • Measures of a Circle

WARM-UP

Facts Practice: 64 Multiplication Facts (Test F)
Mental Math: Count by 6’s from 6 to 60. Count by 60’s from 60 to 360. How many minutes are 2 hours? ... 3 hours? ... 4 hours? ... 10 hours?
a. 2 hours 15 minutes is how many minutes?
b. 2000 – 500
c. $2\frac{1}{3} + 2\frac{1}{3}$
d. $2\frac{1}{2} - 2\frac{1}{2}$
e. How many minutes is 1$\frac{1}{2}$ hours? ... 2$\frac{1}{2}$ hours?
f. Find half of 100, $\div 2$, $\div 5$, $\div 5$, $\times 10$, $\div 5$

Problem Solving:
The numbers 3, 6, 10, and 15 are examples of triangular numbers. The numbers 4, 9, 16, and 25 are examples of square numbers. Find a two-digit number that is both a triangular number and a square number.

NEW CONCEPTS

Perimeter

When line segments enclose an area, a polygon is formed. We can find the distance around a polygon by adding the lengths of all the segments that form the polygon. The distance around a polygon is called the perimeter.

We should note that the word length has more than one meaning. We have used length to mean the measure of a segment. But length may also mean the longer dimension of a rectangle. We use the word width to mean the shorter dimension of a rectangle.

Example 1  What is the perimeter of this rectangle?
**Math 6/5, Lesson 53**

Sample taken from Math 6/5 (Third Edition), page 269

---

**Solution**

The perimeter is the distance around the rectangle. This rectangle has a length of 3 cm and a width of 2 cm. The four sides measure 2 cm, 3 cm, 2 cm, and 3 cm. We add the lengths of the sides and find that the perimeter is **10 cm**.

\[2 \text{ cm} + 3 \text{ cm} + 2 \text{ cm} + 3 \text{ cm} = 10 \text{ cm}\]

A regular polygon has sides equal in length and angles equal in measure. For example, a square is a regular quadrilateral. Below we show some regular polygons.

- regular triangle
- regular quadrilateral
- regular pentagon
- regular hexagon
- regular octagon

If we know the length of one side of a regular polygon, we can find the perimeter of the polygon by multiplying the length of one side by the number of sides.

**Example 2**

What is the perimeter of this regular triangle?

**Solution**

The perimeter is the total of the lengths of the three sides. We can find this by multiplying the length of one side of the regular triangle by 3.

\[3 \times 12 \text{ inches} = 36 \text{ inches}\]

**Measures of a circle**

A circle is a smooth curve. The length of the curve is its circumference. So the circumference of a circle is the perimeter of the circle. The center of the circle is the “middle point” of the area enclosed by the circle. The radius is the distance from the center to the curve. The diameter is the distance across the circle through its center. Thus, the diameter of a circle is twice the radius.
Activity: Measuring Circles

Materials needed:
- various circular objects such as paper plates, cups, wheels, and plastic kitchenware lids
- ruler, cloth tape measure, string, or masking tape
- Activity Sheet 20 (available in Saxon Math 6/5—Homeschool Tests and Worksheets)

Make a list of some circular objects in your home. Measure the diameter, radius, and circumference of each object, and record the results in the table on Activity Sheet 20.

LESSON PRACTICE

Practice set

a. What is the length of this rectangle?

b. What is the width of the rectangle?

c. What is the perimeter of the rectangle?

d. What is the perimeter of this right triangle?

e. What is the perimeter of this square?

f. What do we call the perimeter of a circle?

g. What do we call the distance across a circle through its middle?

h. If the radius of a circle is 6 inches, what is the diameter of the circle?

MIXED PRACTICE

Problem set

1. Atop the beanstalk Jack was excited to discover that the goose had laid 3 dozen golden eggs. Jack took 15 eggs. How many golden eggs were left?

2. There are 13 players on one team and 9 players on the other team. If some of the players from one team join the other team so that the same number of players are on each team, how many players will be on each team?
3. Draw a diagram to illustrate and solve this problem:

If \( \frac{2}{3} \) of the 30 children had blue eyes, how many of the children had blue eyes? What percent of the children had blue eyes?

4. If water is poured from glass to glass until the amount of water in each glass is the same, how many ounces of water will be in each glass?

5. In the number 123,456,789,000, the 2 means which of the following?
   A. 2 billion  
   B. 20 billion  
   C. 200 billion

6. Which factors of 8 are also factors of 12?

7. From the year 1820 to 1890 was how many decades?

8. Use digits to write nineteen million, four hundred ninety thousand.

9. \( 6 + \left( \frac{2}{3} - 2 \right) \)

10. \( \frac{2}{3} - \left( \frac{2}{3} + 2 \right) \)

11. \( 300 \times 200 \)

12. \( 800 \times 70 \)

13. \( 57 = 500 \)

14. \( 5.64 \times 78 \)

15. \( 865 \times 74 \)

16. \( 983 \times 76 \)

17. \( \$63.14 - \$42.87 \)

18. \( 3106 - 875 \)

19. \( \$68.09 - \$43.56 + \$27.18 + \$14.97 \)

20. \( \frac{31.65}{5} \)

21. \( \frac{4218}{6} \)

22. \( 5361 \div 10 \)

23. Counting by tens, 1236 is closest to which number?
   A. 1230  
   B. 1240  
   C. 1200  
   D. 1300
24. What is the length of this rectangle?

25. What is the perimeter of this rectangle?

26. To multiply 35 by 21, Christina thought of 21 as 20 + 1. Show two choices Christina has for multiplying the numbers.

27. Write 2,050,000 in expanded notation.


29. Freddy found the circumference of the soup can to be 8 3/8 inches. Round 8 3/8 inches to the nearest inch.
Prime and Composite Numbers

WARM-UP

Facts Practice: 60 Improper Fractions to Simplify (Test H)
Mental Math:
   a. How many grams equal one kilogram? A pair of shoes weighs about one kilogram. One shoe weighs about how many grams?
   b. 25% of 16   c. 25% of 160   d. \( \frac{3}{4} \) of 16
   e. 25% of $20.00   f. \sqrt{61}, -2, \div 2, -1, \times 2, -5

Problem Solving:
   Find the next three numbers in this Fibonacci sequence:
   \[ 1, 1, 2, 3, 5, 8, 13, __, __, __, ... \]

NEW CONCEPT

We have practiced listing the factors of whole numbers. Some whole numbers have many factors. Other whole numbers have only a few factors. In one special group of whole numbers, each number has exactly two factors.

Below, we list the first ten counting numbers and their factors. Numbers with exactly two factors are prime numbers. Numbers with more than two factors are composite numbers. The number 1 has only one factor and is neither prime nor composite.

<table>
<thead>
<tr>
<th>Number</th>
<th>Factors</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>prime</td>
</tr>
<tr>
<td>2</td>
<td>1, 2</td>
<td>prime</td>
</tr>
<tr>
<td>3</td>
<td>1, 3</td>
<td>prime</td>
</tr>
<tr>
<td>4</td>
<td>1, 2, 4</td>
<td>composite</td>
</tr>
<tr>
<td>5</td>
<td>1, 5</td>
<td>prime</td>
</tr>
<tr>
<td>6</td>
<td>1, 2, 3, 6</td>
<td>composite</td>
</tr>
<tr>
<td>7</td>
<td>1, 7</td>
<td>prime</td>
</tr>
<tr>
<td>8</td>
<td>1, 2, 4, 8</td>
<td>composite</td>
</tr>
<tr>
<td>9</td>
<td>1, 3, 9</td>
<td>composite</td>
</tr>
<tr>
<td>10</td>
<td>1, 2, 5, 10</td>
<td>composite</td>
</tr>
</tbody>
</table>

We often think of a prime number as a number that is not divisible by any other number except 1 and itself. Listing prime numbers will quickly give us a feel for which numbers are prime numbers.
Example 1  The first three prime numbers are 2, 3, and 5. What are the next three prime numbers?

Solution  We list the next several whole numbers after 5. A prime number is not divisible by any number except 1 and itself, so we mark through numbers that are divisible by some other number.

6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18

The numbers that are not marked through are prime numbers. The next three prime numbers after 5 are 7, 11, and 13.

Every number in the shaded part of this multiplication table has more than two factors. So every number in the shaded part is a composite number.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>21</td>
<td>24</td>
<td>27</td>
<td>30</td>
<td>33</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td>24</td>
<td>28</td>
<td>32</td>
<td>36</td>
<td>40</td>
<td>44</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td>35</td>
<td>40</td>
<td>45</td>
<td>50</td>
<td>55</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>36</td>
<td>42</td>
<td>48</td>
<td>54</td>
<td>60</td>
<td>66</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>14</td>
<td>21</td>
<td>28</td>
<td>35</td>
<td>42</td>
<td>49</td>
<td>56</td>
<td>63</td>
<td>70</td>
<td>77</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
<td>40</td>
<td>48</td>
<td>56</td>
<td>64</td>
<td>72</td>
<td>80</td>
<td>88</td>
</tr>
</tbody>
</table>

In this multiplication table prime numbers appear only in the row and column beginning with 1. We have circled the prime numbers that appear in the table. Even if the table were extended, prime numbers would appear only in the row and column beginning with 1.

We can use arrays to illustrate the difference between prime and composite numbers. An array is a rectangular arrangement of numbers or objects in rows and columns. Here we show three different arrays for the number 12:

XXX
XXX
XXX

XXX XXXXXX
XXX XXXXXX
XXX XXXXXX

1 by 12
2 by 6
3 by 4

Twelve is a composite number, which is demonstrated by the fact that we can use different pairs of factors to form arrays for 12. By turning the book sideways, we can actually form three more arrays for 12 (4 by 3, 6 by 2, and 12 by 1), but these arrays use the same factor pairs as the arrays shown above.
For the prime number 11, however, there is only one pair of factors that forms arrays: 1 and 11.

\[ \begin{array}{l}
1 \\
11
\end{array} \]

Example 2  Draw three arrays for the number 16. Use different factor pairs for each array.

Solution  The multiplication table can guide us. We see 16 at $4 \times 4$ and at $2 \times 8$. So we can draw a 4-by-4 array and a 2-by-8 array. Of course, we can also draw a 1-by-16 array.

\[ \begin{array}{ccc}
\times & \times & \times \\
\times & \times & \times \\
\times & \times & \times \\
\times & \times & \times \\
& 2 & 16
\end{array} \]

LESSON PRACTICE

Practice set  a. The first four prime numbers are 2, 3, 5, and 7. What are the next four prime numbers?

b. List all the factors of 21. Is the number 21 prime or composite? Why?

c. Which counting number is neither prime nor composite?

d. Draw two arrays of X’s for the composite number 9. Use different factor pairs for each array.

MIXED PRACTICE

Problem set  1. The local store buys one dozen pencils for 96¢ and sells them for 20¢ each. How much profit does the store make on a dozen pencils?

2. A small car weighs about 1 ton. If its 4 wheels carry the weight evenly, then each wheel carries about how many pounds?

3. List the numbers that are factors of both 8 and 12.

4. The first five prime numbers are 2, 3, 5, 7, and 11. What are the next three prime numbers?
Math 6/5, Lesson 80
Sample taken from Math 6/5 (Third Edition), page 415

5. By what fraction name for 1 should \( \frac{3}{4} \) be multiplied to make \( \frac{9}{12} \)?  
\[
\frac{3}{4} \times \frac{?}{?} = \frac{9}{12}
\]

6. Write a fraction equal to \( \frac{1}{2} \) that has a denominator of 6. Then write a fraction equal to \( \frac{2}{3} \) that has a denominator of 6. What is the sum of the fractions you wrote?

7. Think of a prime number. How many different factors does it have?

8. Arrange these numbers in order from least to greatest:  
\[
\frac{3}{8}, \frac{4}{6}, \frac{3}{8}, \frac{6}{12}, \frac{7}{7}
\]

9. One mile is 1760 yards. How many yards is \( \frac{3}{4} \) mile?

10. XZ is 84 millimeters. XY equals YZ. Find XY.

11. \( \$8.43 + 68\epsilon + \$15 + 5\epsilon \)

12. \( 6.505 - 1.4 \)

13. \( \$12 - 12\epsilon \)

14. \( \$18.07 \times 6 \)

15. \( 6w = \$76.32 \)

16. \( 2^5 \)

17. \( 70 \div 4791 \)

18. Divide 365 by 7. Write the quotient as a mixed number.

19. \( \frac{3}{4} \) of \( \frac{3}{4} \)

20. \( \frac{3}{2} \times \frac{3}{2} \)

21. \( \frac{3}{10} = \frac{?}{100} \)

22. \( \frac{2}{3} + \frac{1\frac{2}{3}}{3} \)

23. \( 5 - \frac{1}{5} \)

24. \( \frac{7}{10} - \frac{7}{10} \)

25. It is evening. What time will be shown by this clock in \( \frac{3}{2} \) hours?
26. The Sun is about 92,956,000 miles from Earth. Which digit in 92,956,000 is in the millions place?

27. The Sun is about 150,000,000 kilometers from Earth. Write that distance in expanded notation using powers of 10.

28. Is the sequence below arithmetic or geometric? Find the next two terms in the sequence.

   2, 4, 8, 16, _____, _____, ...

29. As the coin was tossed, the team captain called, “Heads!” What is the probability that the captain’s guess was correct?

30. The fraction $\frac{4}{5}$ is equivalent to 0.8 and 80%. Write 0.8 and 80% as unreduced fractions.
Simplifying Improper Fractions

WARM-UP

Facts Practice: 40 Fractions to Reduce (Test I)
Mental Math:
Roman numerals:
   a. Write 13 in Roman numerals.
   b. Write VIII in our number system.

Problem Solving:
This table lists the years from 2001 to 2006 and the day of the week on which each year begins. Notice that each year begins one day of the week later than the first day of the previous year until 2005. Since 2004 is a leap year and has an additional day, the year 2005 begins an additional day later. Copy this table and continue it through the year 2015, which begins on a Thursday.

NEW CONCEPT

We have learned two ways to simplify fractions. We have converted improper fractions to whole numbers or mixed numbers, and we have reduced fractions. In some cases we need to use both ways in order to simplify a fraction. Consider the following story:

After the party some pizza was left over. 
There was $\frac{3}{4}$ of a pizza in one box and $\frac{1}{3}$ of a pizza in another box. Altogether, how much pizza was in the two boxes?

1In Lessons 91–105, the Mental Math section “Roman numerals” reviews concepts from Appendix Topic A. We suggest you complete Appendix Topic A before beginning this lesson.
In this story about combining, we add $\frac{3}{4}$ to $\frac{3}{4}$.

$$\frac{3}{4} + \frac{3}{4} = \frac{6}{4}$$

We see that the sum is an improper fraction. To convert an improper fraction to a mixed number, we divide the numerator by the denominator and write the remainder as a fraction.

$$\frac{6}{4} \rightarrow 4\overline{1}\frac{2}{4}$$

The improper fraction $\frac{6}{4}$ is equal to the mixed number $1\frac{2}{4}$. However, $1\frac{3}{4}$ can be reduced.

$$1\frac{2}{4} = 1\frac{1}{2}$$

The simplified answer to $\frac{3}{4} + \frac{3}{4}$ is $1\frac{1}{2}$.

Example 1: Write $\frac{8}{6}$ as a reduced mixed number.

**Solution**  To convert $\frac{8}{6}$ to a mixed number, we divide 8 by 6 and get $1\frac{2}{6}$. Then we reduce $1\frac{2}{6}$ by dividing both terms of the fraction by 2 to get $1\frac{1}{3}$.

<table>
<thead>
<tr>
<th>Convert</th>
<th>Reduce</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{8}{6} = 1\frac{2}{6}$</td>
<td>$1\frac{1}{3}$</td>
</tr>
</tbody>
</table>

Example 2: Add: $1\frac{7}{8} + 1\frac{3}{8}$

**Solution**  We add to get $2\frac{10}{8}$. We convert the improper fraction $\frac{10}{8}$ to $1\frac{2}{8}$ and add it to the 2 to get $3\frac{2}{8}$. Finally, we reduce the fraction to get $3\frac{1}{4}$.

<table>
<thead>
<tr>
<th>Add</th>
<th>Convert</th>
<th>Reduce</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1\frac{7}{8} + 1\frac{3}{8} = 2\frac{10}{8}$</td>
<td>$2\frac{10}{8} = 3\frac{2}{8}$</td>
<td>$3\frac{2}{8} = 3\frac{1}{4}$</td>
</tr>
</tbody>
</table>
LEsson PRACTICE

Practice set* Simplify each fraction or mixed number:

a. \( \frac{6}{4} \)  

b. \( \frac{10}{6} \)  

c. \( \frac{3}{8} \)  

d. \( \frac{10}{4} \)  

e. \( \frac{10}{4} \)  

f. \( \frac{12}{8} \)  

g. \( \frac{14}{8} \)  

h. \( \frac{10}{8} \)  

Perform each indicated operation. Simplify your answers.

i. \( \frac{5}{6} + \frac{5}{6} \)  

j. \( \frac{2}{4} + \frac{3}{4} \)  

k. \( \frac{5}{3} \times \frac{3}{2} \)  

l. Each side of this square is \( \frac{5}{8} \) inches long. What is the perimeter of the square? Show your work.

\[
\text{Perimeter} = 4 \times \frac{5}{8} \text{ in.}
\]

MIXED PRACTICE

Problem set  

1. Two fathoms deep is 12 feet deep. How deep is 10 fathoms?  

2. When Jessica baby-sits, she is paid $6.50 per hour. If she baby-sits Saturday from 10:30 a.m. to 3:30 p.m., how much money will she be paid?  

3. Use digits to write the number one hundred fifty-four million, three hundred forty-three thousand, five hundred fifteen.  

4. (a) How many quarter-mile laps does Jim have to run to complete 1 mile?  

(b) How many quarter-mile laps does Jim have to run to complete 5 miles?  

5. Write a fraction equal to \( \frac{3}{4} \) that has a denominator of 8. Add that fraction to \( \frac{5}{8} \). Remember to convert the answer to a mixed number.  

6. What mixed number names the number of shaded hexagons?  

7. Which segment does not name a radius of this circle?  
   
   A. \( \overline{OR} \)  
   B. \( \overline{OS} \)  
   C. \( \overline{RT} \)  
   D. \( \overline{OT} \)
8. Compare: $\frac{1}{2}$ of 2 ◯ 2 × $\frac{1}{2}$

9. What is the shape of a can of beans?

10. $AB$ is 3.2 cm. $BC$ is 1.8 cm. $CD$ equals $BC$. Find $AD$.

11. $\frac{3}{4} + \frac{1}{4}$

12. $\frac{5}{8} - \frac{3}{8}$

13. $3 \times \frac{3}{8}$

14. $10 - ($1.25 + 35¢$)$

15. $\frac{4.32}{5}$

16. $416 \times 740$

17. $4.51 - (2.3 + 0.65)$

18. $960 \div 8$

19. $80 \overline{)9600}$

20. $5m = $12.00

21. $\frac{5}{2} \times \frac{2}{3}$

22. $\frac{2}{3} \div \frac{1}{3}$

23. $\frac{2}{3} \div \frac{1}{6}$

Use this information to answer problems 24 and 25:

Tyrone fixed his function machine so that when he puts in a 3, a 9 comes out. When he puts in a 6, an 18 comes out. When he puts in a 9, a 27 comes out.

24. Which of the following does Tyrone’s function machine do to the numbers he puts into it?

A. It adds 3.  
B. It multiplies by 3.

C. It adds 9.  
D. It multiplies by 2 and 3.

25. Tyrone put in a number, and a 12 came out. What number did he put in?
26. Assuming that the sequence below repeats with period 3, write the next 5 terms.

\[ 4, 4, 1, 4, 4, \ldots \]

The days of the week are Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, and Saturday. Make a list of the number of letters in each name. Friday, for instance, has 6 letters and Saturday has 8. Refer to your list of numbers to answer problems 27–30.

27. What number is the median?

28. What number is the mode?

29. What is the range?

30. Find the mean and write it as a mixed number.


**Focus on**

**Scale Drawings**

A *scale drawing* is a picture or diagram of a figure that has the same shape as the figure but is a different size. Below is a scale drawing of the bedroom shared by Jane and Alicia. Notice the legend to the right of the picture. It shows that 1 centimeter in the picture represents 4 feet in the actual bedroom. The equivalence $1 \text{ cm} = 4 \text{ ft}$ is called the *scale*.

![Image of bedroom scale drawing](image)

Since 1 cm in the picture represents 4 ft in the actual bedroom, we also know the following relationships:

- 2 cm represents 8 ft (since $2 \times 4 = 8$)
- 3 cm represents 12 ft (since $3 \times 4 = 12$)
- 4 cm represents 16 ft (since $4 \times 4 = 16$)
- 5 cm represents 20 ft (since $5 \times 4 = 20$)

If we measure the picture, we find that it is 5 cm long and 3 cm wide. This means that the actual bedroom is 20 ft long and 12 ft wide.

1. What is the actual distance between the beds?
2. What is the actual length and width of the closet?
3. What is the actual area of the entire room? What is the area if you subtract the area of the closet?

Measurements in the picture may be fractions of centimeters. For example, a measurement of 0.5 cm ($\frac{1}{2} \text{ cm}$) represents $\frac{1}{2}$ of 4 ft. (Remember that the word *of*, when used with fractions, tells us to multiply.)

$$\frac{1}{2} \text{ of } 4 \text{ ft} = \frac{1}{2} \times 4 \text{ ft} = 2 \text{ ft}$$
4. What is the actual length and width of the beds?

5. What is the actual length and width of the desk?

6. What actual length does a measurement of 1 3 \( \frac{1}{4} \) cm represent? Can you identify an object in the picture that is about that long?

Andrew is on the corner of Wilson and 3rd Avenue. His position is marked by the "\( \times \)" on the scale drawing below. Andrew's home is halfway between Taft and Lincoln on 5th Avenue; it is marked by the symbol \( \text{Â} \).

For problems 7–10 below, assume Andrew travels only along the streets shown.

7. How far is Andrew from the movie theater (\( \text{Â} \)) at the corner of Wilson and 6th Avenue?

8. How far is he from the drugstore (\( \text{Â} \)) on the corner of Carter and 3rd Avenue?

9. How far is he from the library (\( \text{Â} \)) on the corner of Carter and 5th Avenue? Describe three different routes he could take that all give the least distance.

10. How far is Andrew from his home?

11. Measure the straight-line distance in inches between Andrew's starting point and the corner of Carter and 5th Avenue. From this measurement, estimate the actual straight-line distance in yards.

A familiar type of scale drawing is a map. On a certain map of New York City, the scale is 6 cm = 1 mi. This means that 6 centimeters on the map represents 1 mile of actual distance.
12. What length on the map corresponds to an actual distance of 3 miles? What length on the map corresponds to an actual distance of $\frac{3}{2}$ mile?

13. What fraction of a mile corresponds to 1 cm on the map? What fraction of a mile is represented by 5 cm?

14. What length on the map represents an actual distance of $2\frac{3}{8}$ miles?

15. Part of the New York City map is shown below. Estimate the actual distance between Frederick Douglass Boulevard and Lenox Avenue as a fraction of a mile. (Use the shortest distance between the two roads.)

![Map of New York City showing streets and distances]

**Extensions**

a. Draw a scale picture of the kitchen in your home. Include the stove, refrigerator, and other important items. Make your scale 1 in. = 2 ft.

b. Obtain a street map of your city or a nearby city. Using the legend on the map, estimate the shortest distance between your home and a park of your choice, using the road system rather than a straight-line distance. Describe the route you chose.

c. We can make scale models of 3-dimensional figures. Model trains and action figures are examples of scale models. Using cardboard and glue or tape, make a scale model of the barn below. Use the scale 1 cm = 4 ft. Note that the front and the back of the barn are pentagons.

![Scale model of a barn]

6 cm = 1 mi
Math 7/6
Table of Contents

Lesson 62, Writing Mixed Numbers as Improper Fractions .............. 40
Lesson 89, Estimating Square Roots .................................. 46
Lesson 104, Algebraic Addition Activity ................................. 52
Investigation 7, The Coordinate Plane ................................. 58


**Math 7/6, Lesson 62**  
Sample taken from Math 7/6 (Fourth Edition), page 331

---

**Lesson 62**  
**Writing Mixed Numbers as Improper Fractions**

**WARM-UP**

**Facts Practice:** 30 Fractions to Reduce (Test G)  
**Mental Math:** Count by 12’s from 12 to 144.  
  a. $5 \times 40$  
  b. $475 + 1200$  
  c. $3 \times 84$  
  d. $\$8.50 + \$2.50$  
  e. $\frac{3}{5}$ of $\$36.00$  
  f. $\frac{22}{10}$  
  g. $6 \times 8, -4, 4, \times 2, +2, \div 6, \div 2$  
  h. Hold your hands one foot apart.

**Problem Solving:**  
The average number of people in each of two rows is 27. If the people are separated into three rows instead of two, what will be the average number of people per row?

---

**NEW CONCEPT**

Here is another story about pies. In this story a mixed number is changed to an improper fraction.

_There were $3 \frac{3}{8}$ pies on the shelf. The restaurant manager asked the server to cut the whole pies into sixths. Altogether, how many slices of pie were there after the server cut the pies?_

We illustrate this story with circles. There were $3 \frac{3}{8}$ pies on the shelf.
The server cut the whole pies into sixths. Each whole pie then had six slices.

The three whole pies contain 18 slices \((3 \times 6 = 18)\). The 5 additional slices from the \(\frac{5}{6}\) of a pie bring the total to 23 slices (23 sixths). This story illustrates that \(\frac{35}{6}\) is equivalent to \(\frac{23}{6}\).

Now we describe the arithmetic for changing a mixed number such as \(\frac{35}{6}\) to an improper fraction. Recall that a mixed number has a whole-number part and a fraction part.

\[
\text{whole number} \quad \begin{array}{c}
3 \\
5 \\
6
\end{array} \quad \text{fraction}
\]

The denominator of the mixed number will also be the denominator of the improper fraction.

\[
\frac{35}{6} = \frac{6}{6}
\]

The denominator indicates the size of the fraction “pieces.” In this case the fraction pieces are sixths, so we change the whole number 3 into sixths. We know that one whole is \(\frac{6}{6}\), so three wholes is \(3 \times \frac{6}{6}\), which is \(\frac{18}{6}\). Therefore, we add \(\frac{18}{6}\) and \(\frac{5}{6}\) to get \(\frac{23}{6}\).

\[
\frac{6}{6} + \frac{6}{6} + \frac{6}{6} + \frac{5}{6} = \frac{23}{6}
\]

\[
\frac{18}{6} + \frac{5}{6} = \frac{23}{6}
\]
Example 1  Write $2\frac{3}{4}$ as an improper fraction.

Solution  The denominator of the fraction part of the mixed number is fourths, so the denominator of the improper fraction will also be fourths.

$$2\frac{3}{4} = \frac{2 \times 4 + 3}{4} = \frac{11}{4}$$

We change the whole number $2$ into fourths. Since $1$ equals $\frac{4}{4}$, the whole number $2$ equals $2 \times \frac{4}{4}$, which is $\frac{8}{4}$. We add $\frac{8}{4}$ and $\frac{3}{4}$ to get $\frac{11}{4}$.

Example 2  Write $5\frac{2}{3}$ as an improper fraction.

Solution  We see that the denominator of the improper fraction will be thirds.

$$5\frac{2}{3} = \frac{5 \times 3 + 2}{3} = \frac{17}{3}$$

Some people use a quick, mechanical method to find the numerator of the improper fraction. Looking at the mixed number, they multiply the denominator by the whole number and then add the numerator. The result is the numerator of the improper fraction.

Example 3  Write $1\frac{2}{3}$ and $2\frac{3}{5}$ as improper fractions. Then multiply the improper fractions. What is the product?

Solution  First we write $1\frac{2}{3}$ and $2\frac{3}{5}$ as improper fractions.

$$1\frac{2}{3} = \frac{3 \times 1 + 2}{3} = \frac{5}{3}$$

$$2\frac{3}{5} = \frac{5 \times 2 + 3}{5} = \frac{12}{5}$$

$$\frac{5}{3} \times \frac{12}{5} = \frac{60}{15} = 4$$
Next we multiply $\frac{5}{3}$ by $\frac{12}{5}$.

$$\frac{5}{3} \times \frac{12}{5} = \frac{60}{15}$$

The result is an improper fraction, which we simplify.

$$\frac{60}{15} = 4$$

So $1\frac{3}{5} \times 2\frac{5}{8}$ equals 4.

**LESSON PRACTICE**

**Practice set** Write each mixed number as an improper fraction:

a. $2\frac{4}{5}$  
   b. $3\frac{1}{2}$  
   c. $1\frac{3}{4}$

d. $6\frac{1}{4}$  
   e. $1\frac{5}{6}$  
   f. $3\frac{3}{10}$

g. $2\frac{1}{3}$  
   h. $12\frac{3}{2}$  
   i. $3\frac{1}{6}$

j. Write $1\frac{3}{5}$ and $3\frac{1}{2}$ as improper fractions. Then multiply the improper fractions. What is the product?

**MIXED PRACTICE**

**Problem set**

1. In music there are whole notes, half notes, quarter notes, and eighth notes.
   
   (a) How many quarter notes equal a whole note?

   (b) How many eighth notes equal a quarter note?

2. Don is 5 feet 2\(\frac{1}{2}\) inches tall. How many inches tall is that?

3. Which of these numbers is not a prime number?
   
   A. 11  
   B. 21  
   C. 31  
   D. 41

4. Write $1\frac{3}{5}$ and $1\frac{1}{3}$ as improper fractions, and multiply the improper fractions. What is the product?

5. If the chance of rain is 20%, what is the chance that it will not rain?

6. The prices for three pairs of skates were $36.25, $41.50, and $43.75. What was the average price for a pair of skates?
7. Instead of dividing 15 by $2\frac{1}{2}$, Solomon doubled both numbers and then divided mentally. What was Solomon's mental division problem and its quotient?

Find each missing number:

8. $m - 4\frac{3}{8} = 3\frac{1}{4}$
9. $n + \frac{3}{10} = \frac{3}{5}$

10. $6d = 0.456$
11. $0.04w = 1.5$

12. $\frac{1}{2} + \frac{3}{4} + \frac{5}{8}$
13. $\frac{5}{6} - \frac{1}{2}$

14. $\frac{1}{2} \cdot \frac{4}{5}$
15. $\frac{2}{3} \div \frac{1}{2}$

16. $1 - (0.2 - 0.03)$
17. $(0.14)(0.16)$

18. One centimeter equals 10 millimeters. How many millimeters does 2.5 centimeters equal?

19. List all of the common factors of 18 and 24. Then circle the greatest common factor.

20. Ten marbles are in a bag. Four of the marbles are red. If one marble is drawn from the bag, what ratio expresses the probability that it will be red?

21. If the perimeter of a square is 40 mm, what is the area of the square?

22. At 6 a.m. the temperature was $-6^\circ F$. At noon the temperature was $14^\circ F$. From 6 a.m. to noon the temperature rose how many degrees?

23. Lisa used a compass to draw a circle with a radius of $1\frac{1}{2}$ inches.

(a) What was the diameter of the circle?

(b) What was the circumference of the circle? (Use 3.14 for $\pi$.)
The circle graph below shows the favorite sports of 100 people. Refer to the graph to answer problems 24–27.

24. How many more people favored baseball than favored football?

25. What fraction of the people favored baseball?

26. Was any sport the favorite sport of the majority of the people surveyed? Write one or two sentences to explain your answer.

27. Since baseball was the favorite sport of 40 out of 100 people, it was the favorite sport of 40% of the people surveyed. What percent of the people answered that football was their favorite sport?

28. What number is 40% of 200?

29. Here we show 18 written as a product of prime numbers:

\[ 2 \cdot 3 \cdot 3 \]

Write 20 as a product of prime numbers.

30. Judges awarded Sandra these scores for her performance on the vault:

\[ 9.1, 8.9, 9.0, 9.2, 9.2 \]

What is the median score?
Lesson 89

Estimating Square Roots

WARM-UP

Facts Practice: Linear Measurement (Test K)
Mental Math: Count up and down by 3's between −15 and 15.
   a. 90 − 90  b. 1000 − 405  c. 6 × $7.99
   d. Double $27.00.  e. 87.5 ÷ 100  f. 20 × 36
   g. 3 × 3, + 2, × 5, − 5, × 2, ÷ 10, + 5, ÷ 5
   h. About how many meters tall is a classroom door?

Problem Solving:
The perimeter of this rectangle is 1 m. What is its length?

21 cm

NEW CONCEPT

We have practiced finding square roots of perfect squares from 1 to 100. In this lesson we will find the square roots of perfect squares greater than 100. We will also use a guess-and-check method to estimate the square roots of numbers that are not perfect squares. As we practice, our guesses will improve and we will begin to see clues to help us estimate.

Example 1
Simplify: \(\sqrt{400}\)

Solution
We need to find a number that, when multiplied by itself, has a product of 400.

\[ \square \times \square = 400 \]

We know that \(\sqrt{400}\) is more than 10, because \(10 \times 10\) equals 100. We also know that \(\sqrt{400}\) is much less than 100, because \(100 \times 100\) equals 10,000. Since \(\sqrt{4}\) equals 2, the 4 in \(\sqrt{400}\) hints that we should try 20.

\[ 20 \times 20 = 400 \]

We find that \(\sqrt{400}\) equals 20.
Example 2  Simplify: \( \sqrt{625} \)

**Solution**  In example 1 we found that \( \sqrt{400} \) equals 20. Since \( \sqrt{625} \) is greater than \( \sqrt{400} \), we know that \( \sqrt{625} \) is greater than 20. We find that \( \sqrt{625} \) is less than 30, because \( 30 \times 30 \) equals 900. Since the last digit is 5, perhaps \( \sqrt{625} \) is 25. We multiply to find out.

\[
\begin{array}{c}
 25 \\
\times 25 \\
\hline
125 \\
50 \\
\hline
625
\end{array}
\]

We find that \( \sqrt{625} \) equals 25.

We have practiced finding the square roots of numbers that are perfect squares. Now we will practice estimating the square root of numbers that are not perfect squares.

Example 3  Between which two consecutive whole numbers is \( \sqrt{20} \)?

**Solution**  Notice that we are not asked to find the square root of 20. To find the whole numbers on either side of \( \sqrt{20} \), we can first think of the perfect squares that are on either side of 20. Here we show the first few perfect squares, starting with 1.

\[
1, 4, 9, 16, 25, 36, 49
\]

We see that 20 is between the perfect squares 16 and 25. So \( \sqrt{20} \) is between \( \sqrt{16} \) and \( \sqrt{25} \).

\[
\sqrt{16}, \sqrt{20}, \sqrt{25}
\]

Since \( \sqrt{16} \) is 4 and \( \sqrt{25} \) is 5, we see that \( \sqrt{20} \) is between 4 and 5.

\[
\begin{array}{c}
\sqrt{16} \\
4 \\
\sqrt{20} \\
5 \\
\sqrt{25}
\end{array}
\]

Using the reasoning in example 3, we know there must be some number between 4 and 5 that is the square root of 20. We try 4.5.

\[4.5 \times 4.5 = 20.25\]

We see that 4.5 is too large, so we try 4.4.

\[4.4 \times 4.4 = 19.36\]
We see that 4.4 is too small. So \( \sqrt{20} \) is greater than 4.4 but less than 4.5. (It is closer to 4.5.) If we continued this process, we would never find a decimal number or fraction that exactly equals \( \sqrt{20} \). This is because \( \sqrt{20} \) belongs to a number family called the **irrational numbers**.

Irrational numbers cannot be expressed exactly as a ratio (that is, as a fraction or decimal). We can only use fractions or decimals to express the **approximate** value of an irrational number.

\[
\sqrt{20} \approx 4.5
\]

Recall from Lesson 47 that the wavy equal sign means “is approximately equal to.” The square root of 20 is approximately equal to 4.5.

**Example 4** Use a calculator to approximate the value of \( \sqrt{20} \) to two decimal places.

**Solution** We clear the calculator and then enter \( \sqrt{20} \) (or \( \sqrt{x} \)). The display will show 4.472135955. The actual value of \( \sqrt{20} \) contains an infinite number of decimal places. The display approximates \( \sqrt{20} \) to nine or so decimal places (depending on the model). We are asked to show two decimal places, so we round the displayed number to 4.47.

**Lesson Practice**

**Practice set** Find each square root:

<table>
<thead>
<tr>
<th>a. ( \sqrt{169} )</th>
<th>b. ( \sqrt{484} )</th>
<th>c. ( \sqrt{961} )</th>
</tr>
</thead>
</table>

Each of these square roots is between which two consecutive whole numbers? Find the answer without using a calculator.

<table>
<thead>
<tr>
<th>d. ( \sqrt{2} )</th>
<th>e. ( \sqrt{15} )</th>
<th>f. ( \sqrt{40} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>g. ( \sqrt{60} )</td>
<td>h. ( \sqrt{70} )</td>
<td>i. ( \sqrt{80} )</td>
</tr>
</tbody>
</table>

Use a calculator to approximate each square root to two decimal places:

<table>
<thead>
<tr>
<th>j. ( \sqrt{3} )</th>
<th>k. ( \sqrt{10} )</th>
<th>l. ( \sqrt{50} )</th>
</tr>
</thead>
</table>

The order of keystrokes depends on the model of calculator. See the instructions for your calculator if the keystroke sequences described in this lesson do not work for you.
MIXED PRACTICE

1. What is the difference when the product of $\frac{3}{5}$ and $\frac{1}{3}$ is subtracted from the sum of $\frac{3}{4}$ and $\frac{1}{4}$?

2. A dairy cow can give 4 gallons of milk per day. How many cups of milk is that (1 gallon = 4 quarts; 1 quart = 4 cups)?

3. The recipe called for $\frac{3}{4}$ cup of sugar. If the recipe is doubled, how much sugar should be used?

4. Draw a ratio box for this problem. Then solve the problem using a proportion.

   The recipe called for sugar and flour in the ratio of 2 to 9. If the chef used 18 pounds of flour, how many pounds of sugar were needed?

5. Which of these numbers is greater than 6 but less than 7?
   A. $\sqrt{65}$  B. $\sqrt{67}$  C. $\sqrt{45}$  D. $\sqrt{76}$

6. Express the missing factor as a mixed number:
   
   $7n = 30$

7. Amanda used a compass to draw a circle with a radius of 4 inches.
   (a) What is the diameter of the circle?
   (b) What is the circumference of the circle?

8. In problem 7 what is the area of the circle Amanda drew?

9. What is the area of the triangle at right?

10. (a) What is the area of this parallelogram?
    (b) What is the perimeter of this parallelogram?
11. Write 0.5 as a fraction and subtract it from $3\frac{1}{2}$. What is the difference?

12. Write $\frac{3}{4}$ as a decimal, and multiply it by 0.6. What is the product?

13. $2 \times 15 + 2 \times 12$

14. $\sqrt{900}$

15. $6 \div 8$

16. $1\frac{3}{5} \times 10 \times \frac{1}{4}$

17. $37\frac{1}{2} \div 100$

18. $3 \div 7\frac{1}{2}$

19. What is the place value of the 7 in 987,654.321?

20. Write the decimal number five hundred ten and five hundredths.

21. $30 + 60 + m = 180$

22. Half of the guests are girls. Half of the girls have brown hair. Half of the brown-haired girls wear their hair long. Of the 32 guests, how many are girls with long, brown hair?

Refer to the pictograph below to answer problems 23–25.

Books Read This Year

<table>
<thead>
<tr>
<th>Johnny</th>
<th>Mary</th>
<th>Pat</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Books" /></td>
<td><img src="image" alt="Books" /></td>
<td><img src="image" alt="Books" /></td>
</tr>
</tbody>
</table>

represents 4 books.

23. How many books has Johnny read?

24. Mary has read how many more books than Pat?

25. Write a question that relates to this graph and answer the question.
26. Solve this proportion: \( \frac{12}{8} = \frac{21}{m} \)

27. The face of this spinner is divided into 12 congruent regions. If the spinner is spun once, what is the probability that it will stop on a 3?

28. If two angles are complementary, and if one angle is acute, then the other angle is what kind of angle?
   A. acute          B. right          C. obtuse

29. Simplify:
   (a) 100 cm + 100 cm (Write the answer in meters.)
   (b) \( \frac{5 \text{ in.}}{2} \)

30. If each small block has a volume of 1 cubic inch, then what is the volume of this cube?
Lesson 104

Algebraic Addition Activity

WARM-UP

Note: Because the New Concept in this lesson takes more time than usual, today’s Warm-Up has been omitted.

NEW CONCEPT

One model for the addition of signed numbers is the number line. Another model for the addition of signed numbers is the electrical-charge model, which is used in the Sign Wars game. In this model signed numbers are represented by positive and negative charges that can neutralize each other.

Activity: Sign Wars Game

In Sign Wars positives “battle” negatives. After each battle we ask ourselves, “Who survived?” and then write our answer. There are four skill levels to the game. Be sure you are successful at one level before moving to the next level.

Level 1 Positive and negative signs are placed randomly on a “screen.” For the battle we neutralize positive and negative pairs by crossing out the signs as shown. (Appropriate sound effects strengthen the experience!)

![Before and After Battle Diagrams]

After the battle we count the remaining positives or negatives to determine who survived. In the battle shown above, there are two positive survivors. See whether you can determine the number and type of survivors for the following practice screens:

![Practice Screens Diagrams]
Level 2  Positives and negatives are displayed in counted clusters. The suggested strategy is to group forces before the battle. So +3 combines with +1 to form +4, and −5 combines with −2 to form −7.

In this battle there were three more negatives than positives, so −3 survived. See whether you can determine the number and type of survivors for the following practice screens:

Level 3  Positive and negative clusters can be displayed with two signs, one sign, or no sign. Clusters appear “in disguise” by taking on an additional sign or by dropping a sign. The first step is to remove the disguise. A cluster with no sign, with “− −,” or with “+ +” is a positive cluster. A cluster with “+ −” or with “− +” is a negative cluster. If a cluster has a “shield” (parentheses), look through the shield to see the sign.

Examples of Positives  Examples of Negatives

\[-(-3) = +3\]  \[-(+2) = -2\]
\[-2 = +2\]  \[+(-3) = -3\]
\[4 = +4\]  \[+-1 = -1\]
\[+1 = +1\]  \[-4 = -4\]

Disguised  Disguises removed

\[3\]  \[+3\]
\[(-5)\]  \[-4\]
\[-(+2)\]  \[-2\]
See whether you can determine the number and type of survivors for the following practice screens:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>+(-5)</td>
<td>-4</td>
</tr>
<tr>
<td>+6</td>
<td>+(+4)</td>
<td>-2</td>
</tr>
</tbody>
</table>

**Level 4** Extend Level 3 to a line of clusters without using a screen. Determine the survivors for this battle:

\[-3 \div (-4) - (-5) - (+2) + (+6)\]

Use the following steps to find the answer:

**Step 1:** Remove the disguises: \(-3 - 4 + 5 - 2 + 6\)

**Step 2:** Group forces: \(-9 + 11\)

**Step 3:** Who survived? \(+2\)

**LESSON PRACTICE**

**Practice set** Simplify:

a. \(-2 + (-3) - (-4) + 5\)

b. \(-3 + (+2) - (+5) - (-6)\)

c. \(+3 - (-4) + 6 + 7 - (-1)\)

d. \(2 + (-3) - (-9) - (+7) + (+1)\)

e. \(3 - (-5) - (-4) + 2 + +8\)

f. \((-10) + (+20) - (-30) + (-40)\)

**MIXED PRACTICE**

**Problem set**

1. A pyramid with a square base has how many more edges than vertices?

2. Becki weighed 7 lb 8 oz when she was born and 12 lb 6 oz at 3 months. How many pounds and ounces did Becki gain in 3 months?
3. The team won 6 games and lost 10. What was its win-
loss ratio?

4. Another team’s win-loss ratio was 3 to 2. If the team
had played 20 games without a tie, how many games
had it won?

5. If Molly tosses a coin and rolls a number cube, what is
the probability of the coin landing heads up and the
number cube stopping with a 6 on top?

6. (a) What is the perimeter of this parallelogram?

(b) What is the area of this parallelogram?

7. If each acute angle of a parallelogram measures 59°, then
what is the measure of each obtuse angle?

8. The center of this circle is the origin.
The circle passes through (2, 0).

(a) Estimate the area of the circle in square units by counting
squares.

(b) Calculate the area of the circle by using 3.14 for π.

9. Which ratio forms a proportion with \( \frac{2}{3} \)?
   A. \( \frac{2}{4} \)   B. \( \frac{3}{4} \)   C. \( \frac{4}{6} \)   D. \( \frac{3}{2} \)

10. Complete this proportion: \( \frac{6}{8} = \frac{a}{12} \)

11. What is the perimeter of the hexagon at right? Dimensions are
    in centimeters.
Complete the table to answer problems 12–14.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{20} )</td>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td>(a)</td>
<td>1.2</td>
<td>(b)</td>
</tr>
<tr>
<td>(a)</td>
<td>(b)</td>
<td>10%</td>
</tr>
</tbody>
</table>

15. Sharon bought a notebook for 40% off the regular price of $6.95. What was the sale price of the notebook?

16. Between which two consecutive whole numbers is \( \sqrt{200} \)?

17. Compare:

\[ \left( \frac{1}{2} \right)^3 \bigcirc \text{the probability of 3 consecutive “heads” coin tosses} \]

18. Divide 0.624 by 0.05 and round the quotient to the nearest whole number.

19. The average of three numbers is 20. What is the sum of the three numbers?

20. Write the prime factorization of 450 using exponents.

21. \(-3 + -5 - -4 - +2\)

22. \(3^4 + 5^2 \times 4 - \sqrt{100} \times 2^3\)

23. How many blocks 1 inch on each edge would it take to fill a shoe box that is 12 inches long, 6 inches wide, and 5 inches tall?

24. Three fourths of the 60 athletes played in the game. How many athletes did not play?

25. The distance a car travels can be found by multiplying the speed of the car by the amount of time the car travels at that speed. How far would a car travel in 4 hours at 88 kilometers per hour?

\[ \frac{88 \text{ km}}{1 \text{ hr}} \times \frac{4 \text{ hr}}{1} \]
26. (a) What is the area of the shaded rectangle?
(b) What is the area of the unshaded rectangle?
(c) What is the combined area of the two rectangles?

27. Kobe measured the circumference and diameter of four circles. Then he divided the circumference by the diameter of each circle to find the number of diameters in a circumference. Here are his answers:

3.12, 3.2, 3.15, 3.1

Find the average of Kobe’s answers. Round the average to the nearest hundredth.

28. Norton was thinking of a two-digit counting number, and he asked Simon to guess the number. Describe how you can find the probability that Simon will guess correctly on the first try.

29. The coordinates of three vertices of a triangle are (3, 5), (−1, 5), and (−1, −3). What is the area of the triangle?

30. \[
\frac{2 \text{ gal}}{1} \times \frac{4 \text{ qt}}{1 \text{ gal}} \times \frac{2 \text{ pt}}{1 \text{ qt}}
\]
Focus on

The Coordinate Plane

By drawing two number lines perpendicular to each other and by extending the unit marks, we can create a grid called a coordinate plane.

The point at which the number lines intersect is called the origin. The horizontal number line is called the x-axis, and the vertical number line is called the y-axis. We graph a point by marking a dot at the location of the point. We can name the location of any point on this coordinate plane with two numbers. The numbers that tell the location of a point are called the coordinates of the point.

The coordinates of a point are written as an ordered pair of numbers in parentheses; for example, (3, -2). The first number is the x-coordinate. It shows the horizontal (→) direction and distance from the origin. The second number, the y-coordinate, shows the vertical (↑) direction and distance from the origin. The sign of the coordinate shows the direction. Positive coordinates are to the right or up, and negative coordinates are to the left or down.
To graph (3, −2), we begin at the origin and move three units to the right along the x-axis. From there we move down two units and mark a dot. We may label the point we graphed (3, −2). On the previous coordinate plane, we have graphed three additional points and identified their coordinates. Notice that each pair of coordinates is different and designates a unique point.

Refer to the coordinate plane below to answer problems 1–6.

1. What are the coordinates of point A?
2. Which point has the coordinates (−1, 3)?
3. What are the coordinates of point E?
4. Which point has the coordinates (1, −3)?
5. What are the coordinates of point D?
6. Which point has the coordinates (3, −1)?

The coordinate plane is useful in many fields of mathematics, including algebra and geometry.

In the next section of this investigation we will designate points on the plane as vertices of rectangles. Then we will calculate the perimeter and area of each rectangle.
Suppose we are told that the vertices of a rectangle are located at 
(3, 2), (–1, 2), (–1, –1), and (3, –1). We graph the points and then 
draw segments between the points to draw the rectangle.

We see that the rectangle is four units long and three units wide. Adding the lengths of the four sides, we find that the 
perimeter is 14 units. To find the area, we can count the 
unit squares within the rectangle. There are three rows of 
four squares, so the area of the rectangle is 3 × 4, which is 
12 square units.

For problems 7–9, use Activity Sheets 10–12 (available in 
Saxon Math 7/6—Homeschool Tests and Worksheets).

7. The vertices of a rectangle are located at (–2, –1), (2, –1), 
(2, 3), and (–2, 3).
   (a) Graph the rectangle. What do we call this special type 
of rectangle?
   
   (b) What is the perimeter of the rectangle?
   
   (c) What is the area of the rectangle?

8. The vertices of a rectangle are located at (–4, 2), (0, 2), 
(0, 0), and (–4, 0).
   (a) Graph the rectangle. Notice that one vertex is located 
at (0, 0). What is the name for this point on the 
coordinate plane?
   
   (b) What is the perimeter of the rectangle?
   
   (c) What is the area of the rectangle?
9. Three vertices of a rectangle are located at (3, 1), (−2, 1), and (−2, −3).
   (a) Graph the rectangle. What are the coordinates of the fourth vertex?
   (b) What is the perimeter of the rectangle?
   (c) What is the area of the rectangle?

As the following activity illustrates, we can use coordinates to give directions for making a drawing.

**Activity: Drawing on the Coordinate Plane**

Materials needed:

- Activity Sheets 13–15 (available in *Saxon Math 7/6—Homeschool Tests and Worksheets*)

10. Christy made this drawing on a coordinate plane. Then she wrote directions for making the drawing. Follow Christy’s directions below to make a similar drawing on your coordinate plane. The coordinates of the vertices are listed in order, as in a “dot-to-dot” drawing.

Draw segments to connect the following points in order:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>(−1, −2)</td>
<td>b.</td>
<td>(−1, −3)</td>
</tr>
<tr>
<td>c.</td>
<td>(−1 1/2, −5)</td>
<td>d.</td>
<td>(−1 1/2, −6)</td>
</tr>
<tr>
<td>e.</td>
<td>(−1, −8)</td>
<td>f.</td>
<td>(−1, −8 1/2)</td>
</tr>
<tr>
<td>g.</td>
<td>(−2, −9 1/2)</td>
<td>h.</td>
<td>(−2, −10)</td>
</tr>
<tr>
<td>i.</td>
<td>(2, −10)</td>
<td>j.</td>
<td>(2, −9 1/2)</td>
</tr>
<tr>
<td>k.</td>
<td>(1, −8 1/2)</td>
<td>l.</td>
<td>(1, −8)</td>
</tr>
<tr>
<td>m.</td>
<td>(1 1/2, −6)</td>
<td>n.</td>
<td>(1 1/2, −5)</td>
</tr>
<tr>
<td>o.</td>
<td>(1, −3)</td>
<td>p.</td>
<td>(1, −2)</td>
</tr>
</tbody>
</table>
Lift your pencil and restart:

a. $(-2\frac{1}{2}, 4)$  
   b. $(2\frac{1}{2}, 4)$  
   c. $(5, -2)$  
   d. $(-5, -2)$  
   e. $(-2\frac{1}{2}, 4)$

11. Jenny wrote the following directions for a drawing. Follow her directions to make the drawing on your own paper. Draw segments to connect the following points in order:

   a. $(-9, 0)$  
   b. $(6, -1)$  
   c. $(8, 0)$  
   d. $(7, 1)$  
   e. $(6, \frac{1}{2})$  
   f. $(6, -1)$  
   g. $(9, -2\frac{1}{2})$  
   h. $(10, -2)$  
   i. $(7, 1)$  
   j. $(6, 1\frac{1}{2})$  
   k. $(-10\frac{1}{2}, 3)$  
   l. $(-11, 2)$  
   m. $(-10\frac{1}{2}, 0)$  
   n. $(-10, -1\frac{1}{2})$  
   o. $(9, -2\frac{1}{2})$  
   p. $(-3, -3\frac{1}{2})$  
   q. $(-7, -8)$  
   r. $(-10, -8)$  
   s. $(-9, -1\frac{1}{2})$

Lift your pencil and restart:

a. $(-10\frac{1}{2}, 0)$  
   b. $(-11, -\frac{1}{2})$  
   c. $(-12, \frac{1}{2})$  
   d. $(-11\frac{1}{2}, 1)$  
   e. $(-12, 1\frac{1}{2})$  
   f. $(-11\frac{1}{2}, 2)$  
   g. $(-12, 2\frac{1}{2})$  
   h. $(-11, 3\frac{1}{2})$  
   i. $(-10\frac{1}{2}, 3)$  
   j. $(-11\frac{1}{2}, 8)$  
   k. $(-9\frac{1}{2}, 8)$  
   l. $(-7, 3)$  
   m. $(-6, 2\frac{1}{2})$  
   n. $(-7, 3)$  
   o. $(-6, 5)$  
   p. $(-4, 5)$  
   q. $(-1, 2)$

12. On a coordinate plane, make a straight-segment drawing. Then write directions for making the drawing by listing the coordinates of the vertices in “dot-to-dot” order.

Note: Problems intended for additional exposure to the concepts in this investigation are available in the appendix.
Math 8/7
Table of Contents

Lesson 35, Adding, Subtracting, Multiplying, and Dividing Decimal Numbers ............................................................. 64
Lesson 65, Ratio Problems Involving Totals ............................................. 71
Lesson 107, Slope ........................................................................ 77
Investigation 5, Creating Graphs ......................................................... 86
Adding, Subtracting, Multiplying, and Dividing Decimal Numbers

WARM-UP

Facts Practice: Measurement Facts (Test H)
Mental Math:

a. $7.50 - $1.99
b. $5 \times 64$
c. $\frac{9}{10} = \frac{7}{50}$
d. Reduce $\frac{11}{12}$
e. $4^2 - \sqrt{4}$
f. $\frac{3}{5}$ of 24
g. Start with the number of inches in two feet, + 1, $\times 4$, $\sqrt{4}$.
What do we call this many years?

Problem Solving:
Copy this problem and fill in the missing digits:

\[
\begin{array}{c}
\underline{8} \\
\underline{16} \\
\underline{24} \\
\underline{32} \\
\end{array}
\]

NEW CONCEPTS

Adding and subtracting decimal numbers is similar to adding and subtracting money. We align the decimal points to ensure that we are adding or subtracting digits that have the same place value.

Example 1 Add: $3.6 + 0.36 + 36$

Solution
We align the decimal points vertically.
A number written without a decimal point is a whole number, so the decimal point is to the right of 36.

\[
\begin{array}{c}
3.6 \\
0.36 \\
+ 36. \\
\hline
39.96 \\
\end{array}
\]
Example 2 Add: 0.1 + 0.2 + 0.3 + 0.4

Solution We align the decimal points vertically and add. The sum is 1.0, not 0.10. Since 1.0 equals 1, we can simplify the answer to 1.

\[
\begin{array}{c}
0.1 \\
0.2 \\
0.3 \\
+ 0.4 \\
\hline
1.0 = 1
\end{array}
\]

Example 3 Subtract: 12.3 − 4.567

Solution We write the first number above the second number, aligning the decimal points. We write zeros in the empty places and subtract.

\[
\begin{array}{c}
12.300 \\
\underline{- 4.567} \\
7.733
\end{array}
\]

Example 4 Subtract: 5 − 4.32

Solution We write the whole number 5 with a decimal point and write zeros in the two empty decimal places. Then we subtract.

\[
\begin{array}{c}
5.000 \\
\underline{- 4.32} \\
0.68
\end{array}
\]

Multiplying decimal numbers

If we multiply the fractions three tenths and seven tenths, the product is twenty-one hundredths.

\[
\frac{3}{10} \times \frac{7}{10} = \frac{21}{100}
\]

Likewise, if we multiply the decimal numbers three tenths and seven tenths, the product is twenty-one hundredths.

\[0.3 \times 0.7 = 0.21\]

Here we use an area model to illustrate this multiplication:

Each side of the square is one unit in length. We multiply three tenths of one side by seven tenths of a perpendicular
side. The product is an area that contains twenty-one hundredths of the square.

\[ 0.3 \times 0.7 = 0.21 \]

Notice that the factors each have one decimal place and the product has two decimal places. **When we multiply decimal numbers, the product has as many decimal places as there are in all the factors combined.**

**Example 5** Multiply: \((0.23)(0.4)\)

**Solution** We need not align decimal points to multiply. We set up the problem as though we were multiplying whole numbers. After multiplying, we count the number of decimal places in both factors. There are a total of three decimal places, so we write the product with three decimal places. We count from right to left, writing one or more zeros in front as necessary. The product of 0.23 and 0.4 is **0.092**.

**Example 6** Multiply: \(35 \times 0.4\)

**Solution** We set up the problem as though we were multiplying whole numbers. After multiplying, we count the total number of decimal places in the factors. Then we place a decimal point in the product so that the product has the same number of decimal places as there are in the factors combined. After placing the decimal point, we simplify the result.

**Example 7** Multiply: \((0.2)(0.3)(0.04)\)

**Solution** Sometimes we can perform the multiplication mentally. First we multiply as though we were multiplying whole numbers: \(2 \cdot 3 \cdot 4 = 24\). Then we count decimal places. There are four decimal places in the three factors. Starting from the right side of 24, we count to the left four places. We write zeros in the empty places.

\[ 0.0024 \]
Dividing decimal numbers Dividing a decimal number by a whole number is similar to dividing money. The decimal point in the answer is straight up from the decimal point in the division box.

Example 8 Divide: $3.425 \div 5$

**Solution** We rewrite the problem with a division box. We place a decimal point in the answer directly above the decimal point in the division box. Then we divide as though we were dividing whole numbers. The answer is 0.685.

$$\begin{array}{c|cc}
5 & 3.425 \\
\hline
5 & 3 & 0 \\
\hline
0 & 42 & 40 \\
\hline
0 & 25 & 25 \\
0 & 0 & 0 \\
\end{array}$$

Example 9 Divide: $0.0144 \div 8$

**Solution** We place the decimal point in the answer directly above the decimal point inside the division box. We write a digit in every place following the decimal point until the division is completed. If we cannot perform a division, we write a zero in that place. The answer is 0.0018.

$$\begin{array}{c|cc}
8 & 0.0144 \\
\hline
8 & 8 & 64 \\
\hline
0 & 64 & 0 \\
\end{array}$$

Example 10 Divide: $1.2 \div 5$

**Solution** We do not write a decimal division answer with a remainder. Since a decimal point fixes place values, we may write a zero in the next decimal place. This zero does not change the value of the number, but it does let us continue dividing. The answer is 0.24.

$$\begin{array}{c|cc}
5 & 1.2 \\
\hline
5 & 1 & 0 \\
\hline
20 & 20 & 0 \\
\end{array}$$

**LESSON PRACTICE**

<table>
<thead>
<tr>
<th>Practice set*</th>
<th>Simplify:</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>$1.2 + 3.45 + 23.6$</td>
</tr>
<tr>
<td>b.</td>
<td>$4.5 + 0.51 + 6 + 12.4$</td>
</tr>
<tr>
<td>c.</td>
<td>$0.2 + 0.4 + 0.6 + 0.8$</td>
</tr>
<tr>
<td>d.</td>
<td>$36.274 - 5.39$</td>
</tr>
<tr>
<td>e.</td>
<td>$16.7 - 1.936$</td>
</tr>
<tr>
<td>f.</td>
<td>$12 - 0.875$</td>
</tr>
</tbody>
</table>
Math 8/7, Lesson 35
Sample taken from Math 8/7 (Third Edition), page 239

MIXED PRACTICE

Problem set

1. During the first six months of the year, the Montgomeries' monthly electric bills were $128.45, $131.50, $112.30, $96.25, $81.70, and $71.70. How can the Montgomeries find their average monthly electric bill for the first six months of the year?

2. There were $2\frac{1}{2}$ gallons of milk in the refrigerator before breakfast. There were $1\frac{3}{4}$ gallons after dinner. How many gallons of milk were consumed during the day?

3. A one-year subscription to a monthly magazine costs $15.60. The regular newsstand price is $1.75 per issue. How much is saved per issue by paying the subscription price?

4. Carlos ran one lap in 1 minute 3 seconds. Orlando ran one lap 5 seconds faster than Carlos. How many seconds did it take Orlando to run one lap?

5. The perimeter of the square equals the perimeter of the regular pentagon. Each side of the pentagon is 16 cm long. How long is each side of the square?

6. Diagram this statement. Then answer the questions that follow.

   Two ninths of the 54 fish in the tank were guppies.

   (a) How many of the fish were guppies?
   (b) How many of the fish were not guppies?
7. A 6-by-6-cm square is cut from a 10-by-10-cm square sheet of paper as shown below. Refer to this figure to answer (a)–(c):

(a) What was the area of the original square?
(b) What was the area of the square that was cut out?
(c) What is the area of the remaining figure?

8. (a) In the square at right, what fraction is not shaded?
(b) What decimal part of the square is not shaded?
(c) What percent of the square is not shaded?

9. The coordinates of three vertices of a rectangle are (–3, 2), (3, –2), and (–3, –2).
   (a) What are the coordinates of the fourth vertex?
   (b) What is the area of the rectangle?

10. (a) Use words to write 100.075.
    (b) Use digits to write the decimal number twenty-five hundred-thousandths.

11. Find the length of this segment

(a) in centimeters.
(b) in millimeters.

12. Miss Edwards bought 11.92 gallons of gasoline at $1.49\frac{9}{10}$ per gallon. Estimate how much she paid for the gasoline.
13. What decimal number names the point marked with an arrow on this number line?

14. This figure illustrates the multiplication of which two decimal numbers? What is their product?

15. What decimal number is halfway between 1.2 and 1.3?

Solve:

16. $15x = 9 \cdot 10$

17. $f + 4.6 = 5.83$

18. $8y = 46.4$

19. $w - 3.4 = 12$

Simplify:

20. $3.65 + 0.9 + 8 + 15.23$

21. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$

22. $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}$

23. $\frac{1}{6} - \left( \frac{1}{2} + \frac{1}{3} \right)$

24. $3\frac{1}{12} - 1\frac{3}{4}$

25. $1.2 \div 10$

26. $(0.3)(0.4)(0.5)$

27. $\left(3\frac{1}{2} + 1\frac{3}{4}\right) \div \left(4 - 3\frac{1}{8}\right)$

For problems 28 and 29, record an estimated answer and an exact answer.

28. $36.45 - 4.912$

29. $4.2 \times 0.9$

30. Use a protractor to draw a triangle that has two 45° angles.
Lesson 65

Ratio Problems Involving Totals

WARM-UP

Facts Practice: Metric Conversions (Test M)

Mental Math:

a. $0.42 \times 50$

b. $1.25 \times 10^{-1}$

c. $\frac{3}{4} = \frac{15}{20}$

d. Convert 0.75 m to mm.

e. $5^3 - 10^2$

f. $\frac{5}{15}$ of $4.00$

g. What is the total cost of a $20.00 item plus 7% sales tax?

Problem Solving:

Copy this problem and fill in the missing digits:

\[ \frac{91\frac{1}{2}}{\underline{\quad \quad}} \]

\[ \underline{\quad \quad} \]

\[ \underline{\quad \quad} \]

NEW CONCEPT

Some ratio problems require that we use the total to solve the problem. Consider the following problem:

*The ratio of boys to girls at the concert was 5 to 4. If there were 180 children at the concert, how many girls were there?*

We begin by making a ratio box. This time we add a third row for the total number of children.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Actual Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>B</td>
</tr>
<tr>
<td>Girls</td>
<td>G</td>
</tr>
<tr>
<td>Total</td>
<td>180</td>
</tr>
</tbody>
</table>

In the ratio column we wrote 5 for boys and 4 for girls, then added these to get 9 for the total ratio number. We were given 180 as the actual count of children. This is a total. We can use two rows from this table to write a proportion. Since we were asked to find the number of girls, we will use the “girls” row.
Because we know both total numbers, we will also use the “total” row. Using these numbers, we solve the proportion.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Actual Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>B</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Girls</td>
<td>G</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>180</td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

\[
\frac{4}{9} = \frac{G}{180}
\]

\[
9G = 720
\]

\[
G = 80
\]

We find there were 80 girls at the concert. We can use this answer to complete the ratio box.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Actual Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Girls</td>
<td>80</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>180</td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

**Example**

The ratio of football players to soccer players in the room was 5 to 7. If the football and soccer players in the room totaled 48, how many were football players?

**Solution**

We use the information in the problem to form a table. We include a row for the total number of players. The total ratio number is 12.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Actual Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Football players</td>
<td>F</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Soccer players</td>
<td>S</td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Total players</td>
<td>48</td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

\[
5 = \frac{F}{12}
\]

\[
12F = 240
\]

\[
F = 20
\]

To find the number of football players, we write a proportion from the “football players” row and the “total players” row. We solve the proportion to find that there were 20 football players in the room. From this information we can complete the ratio box.
LESSON PRACTICE

Practice set  Solve these problems. Begin by drawing a ratio box.

a. Acrobats and clowns converged on the center ring in the ratio of 3 to 5. If a total of 72 acrobats and clowns performed in the center ring, how many were clowns?

b. The ratio of young men to young women at the ball was 8 to 9. If 240 young men were in attendance, how many young people attended in all?

MIXED PRACTICE

Problem set  1. If 5 pounds of apples cost $2.40, then
(a) what is the price per pound?
(b) what is the cost for 8 pounds of apples?

2. (a) Simplify and compare:
\[(0.3)(0.4) + (0.3)(0.5) \bigcirc 0.3(0.4 + 0.5)\]
(b) What property is illustrated by this comparison?

3. Use a ratio box to solve this problem. The ratio of big fish to little fish in the pond was 4 to 11. If there were 1320 fish in the pond, how many big fish were there?

4. The car traveled 350 miles on 15 gallons of gasoline. The car averaged how many miles per gallon? Round the answer to the nearest tenth.

5. The average of 2 and 4 is 3. What is the average of the reciprocals of 2 and 4?

6. Write 12 billion in scientific notation.

7. Diagram this statement. Then answer the questions that follow.

   One sixth of the five dozen eggs were cracked.

   (a) How many eggs were not cracked?
   (b) What was the ratio of eggs that were cracked to eggs that were not cracked?
   (c) What percent of the eggs were cracked?
8. (a) Draw segment $AB$. Draw segment $DC$ parallel to segment $AB$ but not the same length. Draw segments between the endpoints of segments $AB$ and $DC$ to form a quadrilateral.

(b) What type of quadrilateral was formed in (a)?

9. Find the area of each triangle. Dimensions are in centimeters.

(a) $\begin{array}{c}
5 \\
4 \\
\hline
6 \\
\end{array}$

(b) $\begin{array}{c}
4 \\
7.2 \\
\hline
6 \\
\end{array}$

(c) $\begin{array}{c}
9 \\
4 \\
\hline
6 \\
\end{array}$

10. What is the average of the two numbers indicated by arrows on the number line below?

Write equations to solve problems 11 and 12.

11. What number is 75 percent of 64?

12. What is the tax on a $7.40 item if the sales-tax rate is 8%?

13. Find each sum:

(a) $(-3) + (-8)$

(b) $(+3) + (-8)$

(c) $(-3) + (+8) + (-5)$

14. A circle is drawn on a coordinate plane with its center at the origin. One point on the circle is $(3, 4)$. Use a compass and graph paper to graph the circle. Then answer (a) and (b).

(a) What are the coordinates of the points where the circle intersects the x-axis?

(b) What is the diameter of the circle?
15. Use a unit multiplier to convert 0.95 liters to milliliters.

16. Evaluate: \( ab + a + \frac{a}{b} \) if \( a = 5 \) and \( b = 0.2 \)

17. How many small blocks were used to build this cube?

18. Recall that one angle is the complement of another angle if their sum is \( 90^\circ \), and that one angle is the supplement of another if their sum is \( 180^\circ \). In this figure, (a) which angle is a complement of \( \angle BOC \) and (b) which angle is a supplement of \( \angle BOC \)?

19. Round each number to the nearest whole number to estimate the product of 19.875 and \( 4\frac{7}{8} \).

20. Refer to \( \triangle ABC \) at right to answer the following questions:
   (a) What is the measure of \( \angle A \)?
   (b) Which side of the triangle is the longest side?
   (c) Triangle \( ABC \) is an acute triangle. It is also what other type of triangle?
   (d) Triangle \( ABC \)'s line of symmetry passes through which vertex?

21. (a) Describe how to find the median of this set of 12 scores.
    \[ 18, 17, 15, 20, 16, 14, 15, 16, 17, 18, 16, 19 \]
    (b) What is the median of the set of scores?
22. Answer true or false:
   (a) All equilateral triangles are congruent.
   (b) All equilateral triangles are similar.

23. The bar was raised from 2.15 meters to 2.2 meters. How many centimeters was the bar raised?

Simplify:

24. \( \frac{10^3 \cdot 10^2}{10^2} \)

25. \( \frac{4 \text{ days } 5 \text{ hr } 15 \text{ min}}{-1 \text{ days } 7 \text{ hr } 50 \text{ min}} \)

26. \( 4.5 \div (0.4 + 0.5) \)

27. \( \frac{3 + 0.6}{3 - 0.6} \)

28. \( 4\frac{1}{3} \div \left( \frac{1}{6} \cdot 3 \right) \)

29. \( 3^2 + \sqrt{4} \cdot 7 - 3 \)

30. \( |-3| + 4[(5 - 2)(3 + 1)] \)
Math 8/7, Lesson 107
Sample taken from Math 8/7 (Third Edition), page 742

LESSON 107
Slope

WARM-UP

Facts Practice: Percent-Decimal-Fraction Equivalents (Test Q)

Mental Math:

a. \(11000 \text{ (base 2)}\)

b. \(DCCC\)

c. \((-2.5)(-4)\)

d. \((2.5 \times 10^5)^2\)

e. \(2x - 1\frac{1}{2} = 4\frac{1}{2}\)

f. Convert \(-50^\circ C\) to degrees Fahrenheit.

g. 75% of $60

h. 75% more than $60

i. \(7 \times 8, -1, \div 5, \times 3, + 2, + 5, \times 7, + 1, \times 2, - 1, \div 3, + 3, \sqrt{}\)

Problem Solving:

In the 3-by-3 square at right, we can find nine 1-by-1 squares, four 2-by-2 squares, and one 3-by-3 square. Find the total number of squares of any size in the 4-by-4 square.

NEW CONCEPT

Below are the graphs of two functions. The graph of the function on the left indicates the number of feet that equal a given number of yards. Changing the number of yards by one changes the number of feet by three. The graph of the function on the right shows the inverse relationship, the number of yards that equal a given number of feet. Changing the number of feet by one changes the number of yards by one third.

Yards to Feet

![Graph of Yards to Feet]

Feet to Yards

![Graph of Feet to Yards]

Notice that the graph of the function on the left has a steep upward slant going from left to right, while the graph of the function on the right also has an upward slant but is not as
steep. The “slant” of the graph of a function is called its slope. We assign a number to a slope to indicate how steep the slope is and whether the slope is upward or downward. If the slope is upward, the number is positive. If the slope is downward, the number is negative. If the graph is horizontal, the slope is neither positive nor negative; it is zero. If the graph is vertical, the slope cannot be determined.

**Example 1** State whether the slope of each line is positive, negative, zero, or cannot be determined.

(a) \[ y = x - 2 \]

(b) \[ y = 3 \]

(c) \[ 2y + x = 0 \]

(d) \[ x = 3 \]

**Solution** To determine the sign of the slope, follow the graph of the function with your eyes from left to right as though you were reading.

(a) From left to right, the graphed line rises, so the slope is positive.

(b) From left to right, the graphed line does not rise or fall, so the slope is zero.

(c) From left to right, the graphed line slopes downward, so the slope is negative.

(d) There is no left to right component of the graphed line, so we cannot determine if the line is rising or falling. The slope is not positive, not negative, and not zero. The slope of a vertical line cannot be determined.
To determine the numerical value of the slope of a line, it is helpful to draw a right triangle using the background grid of the coordinate plane and a portion of the graphed line. First we look for points where the graphed line crosses intersections of the grid. We have circled some of these points on the graphs below.

Next we select two points from the graphed line and, following the background grid, sketch the legs of a right triangle so that the legs intersect the chosen points. (It is a helpful practice to first select the point to the left and draw the horizontal leg to the right. Then draw the vertical leg.)

We use the words **run** and **rise** to describe the two legs of the right triangle. The **run** is the length of the horizontal leg, and the **rise** is the length of the vertical leg. We assign a positive sign to the rise if it goes up to meet the graphed line and a negative sign if it goes down to meet the graphed line. In the graph on the left, the run is 2 and the rise is +3. In the graph
on the right, the run is 2 and the rise is $-1$. We use these numbers to write the slope of each graphed line.

So the slopes of the graphed lines are these ratios:

\[
\frac{\text{rise}}{\text{run}} = \frac{+3}{2} = \frac{3}{2} \quad \frac{\text{rise}}{\text{run}} = \frac{-1}{2} = -\frac{1}{2}
\]

The slope of a line is the ratio of its rise to its run ("rise over run").

\[
\text{slope} = \frac{\text{rise}}{\text{run}}
\]

A line whose rise and run have equal values has a slope of 1. A line whose rise has the opposite value of its run has a slope of $-1$.

A line that is steeper than the lines above has a slope either greater than 1 or less than $-1$. A line that is less steep than the lines above has a slope that is between $-1$ and 1.

Example 2  Find the slope of the graphed line below.
Solution  We note that the slope is positive. We locate and select two points where the graphed line passes through intersections of the grid. We choose the points (0, −1) and (3, 1). Starting from the point to the left, (0, −1), we draw the horizontal leg to the right. Then we draw the vertical leg up to (3, 1).

\[ \text{Slope} = \frac{2}{3} \]

Note that we could have chosen the points (−3, −3) and (3, 1). Had we done so, the run would be 6 and the rise 4. However, the slope would be the same because \(\frac{4}{6}\) reduces to \(\frac{2}{3}\).

One way to check the calculation of a slope is to “zoom in” on the graph. When the horizontal change is one unit to the right, the vertical change will equal the slope. To illustrate this, we will zoom in on the square just below and to the right of the origin on this graph. This method is a check for reasonableness of calculated slopes and can help prevent mistakes such as inverted slopes.
Activity: Slope

Materials needed:
- Activity Sheet 8 (available in Saxon Math 8/7—Homeschool Tests and Worksheets)

Calculate the slope (rise over run) of each graphed line on the activity sheet by drawing right triangles.

LESSON PRACTICE

Practice set

a. Find the slopes of the “Yards to Feet” and the “Feet to Yards” graphs at the beginning of this lesson.

b. Find the slopes of graphs (a) and (c) in example 1.

c. Mentally calculate the slope of each graphed line below by counting the run and rise rather than by drawing right triangles.

d. For each unit of horizontal change to the right on the graphed lines above, what is the vertical change?
MIXED PRACTICE

Problem set

1. The shirt regularly priced at $21 was on sale for \(\frac{1}{3}\) off. What was the sale price?

2. Nine hundred seventy-five billion is how much less than one trillion? Write the answer in scientific notation.

3. What is the (a) range and (b) mode of this set of numbers?

\[16, 6, 8, 17, 14, 16, 12\]

Use ratio boxes to solve problems 4–6.

4. Riding her bike from home to the lake, Sonia averaged 18 miles per hour (per 60 minutes). If it took her 40 minutes to reach the lake, how far did she ride?

5. The ratio of earthworms to cutworms in the garden was 5 to 2. If there were 140 earthworms and cutworms in the garden, how many were earthworms?

6. The average cost of a new car increased 8 percent in one year. Before the increase the average cost of a new car was $16,550. What was the average cost of a new car after the increase?

7. The points (3, −2), (−3, −2), and (−3, 6) are the vertices of a right triangle. Find the perimeter of the triangle.

8. In this figure, \(\angle ABC\) is a right angle.

(a) Find \(m\angle ABD\).

(b) Find \(m\angle DBC\).

(c) Find \(m\angle BCD\).

(d) Which triangles in this figure are similar?

Write equations to solve problems 9–11.

9. Sixty is 125 percent of what number?

10. Sixty is what percent of 25?
11. Sixty is four more than twice what number?

12. In a can are 100 marbles: 10 yellow, 20 red, 30 green, and 40 blue.
   (a) If a marble is drawn from the can, what is the chance that the marble will not be red?
   (b) If the first marble is not replaced and a second marble is drawn from the can, what is the probability that both marbles will be yellow?

13. Complete the table.

<table>
<thead>
<tr>
<th>FRACTION</th>
<th>DECIMAL</th>
<th>PERCENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{5}{6} )</td>
<td>(a)</td>
<td>(b)</td>
</tr>
</tbody>
</table>

14. Compare: \((x - y)^2 \bigcirc (y - x)^2\) if \(x > y\)

15. Multiply. Write the product in scientific notation.
   \((1.8 \times 10^7)(9 \times 10^{-6})\)

16. (a) Between which two consecutive whole numbers is \(\sqrt{600}\)?
   (b) What are the two square roots of 10?

17. Find three \(x, y\) pairs for the function \(y = x + 1\).
   (a) Graph these number pairs on a coordinate plane and draw a line through the points.
   (b) What is the slope of the graphed line?

18. If the radius of this circle is 6 cm, what is the area of the shaded region?

19. Find the surface area of this rectangular solid. Dimensions are in inches.
20. Find the volume of this right circular cylinder. Dimensions are in centimeters.

21. Find the total surface area of the cylinder in problem 20.

22. The polygon $ABCD$ is a rectangle. Find $m\angle x$.

23. Find the slope of the graphed line:

24. Solve for $x$ in each literal equation:
   (a) $x - y = z$  
   (b) $w = xy$

   Solve:
   
   25. $\frac{0}{21} = \frac{1.5}{7}$
   26. $6x + 5 = 7 + 2x$

   Simplify:
   27. $62 + 5[20 - [4^2 + 3(2 - 1)]]$
   28. $\frac{(6x^2y)(2xy)}{4xy^2}$
   29. $\frac{5\frac{1}{6} + 3.5 - \frac{1}{3}}{2}$
   30. $\frac{(-3)(2)(-4) + (-2)(-3)}{-6}$
Focus on

Creating Graphs

Recall from Investigation 4 that we considered a stem-and-leaf plot that a counselor created to display student test scores. If we rotate that plot 90°, the display resembles a vertical bar graph, or histogram.

A histogram is a special type of bar graph that displays data in equal-sized intervals. There are no spaces between the bars. The height of the bars in this histogram show the number of test scores in each interval.

Histograms and other bar graphs are useful for showing comparisons, but sometimes the visual effect can be misleading. When viewing a graph, it is important to carefully note the scale. Compare these two bar graphs that display the same information.

2. Which of the two graphs visually exaggerates the growth in sales from one year to the next? How was the exaggerated visual effect created?

3. Larry made the bar graph below that compares his test score to Moe’s test score. Create another bar graph that shows the same information in a less misleading way.

Changes over time are often displayed by line graphs. A double-line graph may compare two performances over time. The graph below illustrates the differences in the growing value of a $1000 investment compounded at 7% and at 10% annual interest rates.
4. Create a double-line graph using the information in the table below. Label the axes; then select and number the scales. Make a legend (or key) so that the reader can distinguish between the two graphed lines.

<table>
<thead>
<tr>
<th>First Trade Of</th>
<th>XYZ Corp</th>
<th>ZYX Corp</th>
</tr>
</thead>
<tbody>
<tr>
<td>1993</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>1994</td>
<td>36</td>
<td>28</td>
</tr>
<tr>
<td>1995</td>
<td>34</td>
<td>36</td>
</tr>
<tr>
<td>1996</td>
<td>46</td>
<td>40</td>
</tr>
<tr>
<td>1997</td>
<td>50</td>
<td>46</td>
</tr>
<tr>
<td>1998</td>
<td>50</td>
<td>42</td>
</tr>
</tbody>
</table>

A circle graph (or pie graph) is commonly used to show components of a budget. The entire circle, 100%, may represent monthly income. The sectors of the circle show how the income is allocated.

We see that the sector labeled "food" is 20% of the area of the circle, representing 20% of the income. To make a 20% sector, we could draw a central angle that measures 20% of 360°.

\[
20\% \text{ of } 360° \quad 0.2 \times 360° = 72°
\]

With a protractor we can draw a central angle of 72° to make a sector that is 20% of a circle.
5. Create a pie graph for the table below to show how Kerry spends a weekday. First calculate the number of degrees in the central angle for each sector of the pie graph. Next use a compass to draw a circle with a radius of about $2\frac{1}{2}$ inches. Then, with a protractor and straightedge, divide the circle into sectors of the correct size and label each sector.

<table>
<thead>
<tr>
<th>Activity</th>
<th>% of Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Studies</td>
<td>25%</td>
</tr>
<tr>
<td>Recreation</td>
<td>10%</td>
</tr>
<tr>
<td>Music lessons</td>
<td>10%</td>
</tr>
<tr>
<td>Eating</td>
<td>10%</td>
</tr>
<tr>
<td>Sleeping</td>
<td>40%</td>
</tr>
<tr>
<td>Other</td>
<td>5%</td>
</tr>
</tbody>
</table>

**Extensions**

a. Create a circle graph showing the percentages of your friends and family with various eye colors.

b. Explore the graph-creating capabilities of database computer programs.
Algebra 1/2
Table of Contents

Lesson 59. Proportions with Fractions ......................... 91
Lesson 75, Implied Ratios ........................................ 94
Lesson 105, Evaluating Powers of Negative Bases ............. 98
LESSON 59  Proportions with Fractions

There is no change in the method of solving conditional proportions when they contain fractions or mixed numbers. The first step is to cross multiply. Then we divide or multiply as required to complete the solution.

example 59.1  Solve:  \( \frac{2}{3} = \frac{8}{5} \)

solution  As the first step, we cross multiply.

\[
\frac{2}{3} \cdot \frac{1}{5} = \frac{5}{8} x \quad \text{cross multiplied}
\]

\[
\frac{2}{15} = \frac{5}{8} x \quad \text{simplified}
\]

We finish by multiplying both sides by \( \frac{1}{x} \).

\[
\frac{8}{5} \cdot \frac{2}{15} = \frac{8}{8} x \cdot \frac{8}{8} \quad \text{multiplied both sides by} \ \frac{8}{5}
\]

\[
\frac{16}{75} = x \quad \text{simplified}
\]
example 59.2 Solve: \( \frac{x}{3} = \frac{3}{2} \)

\[
\begin{align*}
\frac{2}{5} \cdot \frac{3}{2} & = \frac{1}{3} \cdot \frac{2}{4} & \text{cross multiplied} \\
\frac{2}{5} \cdot \frac{3}{2} & = \frac{1}{4} & \text{simplified}
\end{align*}
\]

We finish by multiplying both sides by \( \frac{5}{2} \):

\[
\frac{5}{2} \cdot \frac{2}{5} \cdot \frac{3}{2} = \frac{1}{4} \cdot \frac{5}{2}
\]

\[
x = \frac{5}{8}
\]

simplified

practice Solve:

\[
a. \quad \frac{2}{1} = \frac{4}{x}
\]

\[
b. \quad \frac{y}{2} = \frac{5}{1} \quad \frac{1}{3} \quad \frac{6}{b}
\]

\[
c. \quad \frac{1}{2} = \frac{2}{x}
\]

\[
d. \quad \frac{3}{x} = \frac{5}{7}
\]

problem set 59

1. (c) The average of the first three numbers was 42. The average of the next seven numbers was only 12. What was the average of all ten numbers?

2. (c) Seven big ones cost $280,000. Write the two rates (ratios) implied by this statement. How many big ones could be purchased for $120,000?

3. (c) The first one weighed one hundred forty thousand, twenty-six pounds. The second weighed only one hundred thirty-two thousand, seven hundred eighty-one pounds. The first one weighed how many pounds more than the second one?

4. (c) Carolyn could walk 10 miles in 3 hours.
   (a) What was her speed?
   (b) How long would it take her to walk 25 miles at the same speed?

5. (c) Complete the table. Begin by inserting the reference numbers.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(b)</td>
<td>16%</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>(c)</td>
<td>(d)</td>
</tr>
</tbody>
</table>

6. (c) (a) Write 0.093 in scientific notation.
   (b) Write \( 1.2 \times 10^6 \) in standard notation.
7. Nine tenths of what number is 72?

8. What fraction of 6\frac{1}{2} is 8\frac{1}{2}?

9. What decimal part of 630 is 441?

10. Eight and one fourth of what number is 7\frac{1}{4}?

11. One and one fourth of 8\frac{3}{4} is what number?

12. What is the volume of a right solid whose base is the figure shown on the left and whose height is 4 inches? Dimensions are in inches.

\[
\begin{align*}
\text{Base: } & \quad \begin{array}{c}
20 \\
7 \\
46 \\
8
\end{array} \\
\text{Height: } & \quad 4
\end{align*}
\]

13. (a) What is 1\% of 192?

(b) What is 45\% of 192?

14. Find the surface area of this rectangular solid. Dimensions are in meters.

\[
\begin{align*}
\text{Dimensions: } & \quad \begin{array}{c}
4 \\
5 \\
10
\end{array}
\end{align*}
\]

Solve:

15. \(\frac{5}{1} \times \frac{10}{4} = \frac{x}{9}\)

16. \(\frac{\frac{2}{3}}{5} = \frac{12}{p}\)

17. \(\frac{\frac{3}{5} \times 6}{x} = \frac{9}{4}\)

18. \(\frac{\frac{1}{4} \times 5}{p} = 6\)

Simplify:

19. \(49 - 2[(5 - 2)^2 + 4] - 5\)

20. \(\frac{\frac{3}{8} + 2\frac{2}{3} \times (2^3 - 5) - 4}{\frac{11}{4}}\)

21. \(2 \frac{1}{3} \times \frac{11}{12} - 12\)

22. \(\frac{14}{3} - \frac{7}{8} + \frac{11}{48}\)

23. \(171.6 \times 0.007\)

24. \(1171.61 - 13.321\)

25. \(\frac{611.51}{0.03}\)

26. \(\frac{\frac{1}{3} + \frac{1}{2}}{\frac{5}{36}}\)

27. \(\frac{\sqrt{5} \left( \frac{1}{4} - \frac{1}{8} \right)}{2 \frac{7}{8}}\)

28. \(\frac{\frac{6}{3}}{\frac{2}{4}}\)

29. \(6 \frac{1}{4} + 3 \frac{2}{3} \times \frac{1}{2} \times \frac{1}{4} + \frac{1}{8}\)

30. Evaluate: \(x^y + 3xy^2 + 3x^2y + y^4\) if \(x = 1\) and \(y = 2\)
Lesson 75  Implied Ratios

Remember that a ratio is a comparison of two numbers. Ratios are often written in the form of fractions. Remember also that a proportion is a statement that two ratios are equal. These are equal ratios.

\[
\frac{3}{4} = \frac{9}{12}
\]

The equation is called a proportion. Many ratio word problems do not actually use the word ratio. When we read the problem, we must recognize that the problem is a ratio problem. We must also be able to pick out the implied ratio.

example 75.1 It takes \(2\frac{1}{2}\) eggs to make 140 cookies. Jenny wants to make 1680 cookies. How many eggs does she need?

solution This problem is a ratio problem about eggs and cookies. We decide to put eggs in the numerator.

\[
\frac{E}{C} = \frac{E}{C}
\]

The first sentence in the problem gives us the implied ratio. It tells us that the ratio of eggs to cookies is \(2\frac{1}{2}\) to 140. We make the substitution.

\[
\frac{2\frac{1}{2}}{140} = \frac{E}{C}
\]
Jenny wants to make 1680 cookies, so we use 1680 for $C$ and solve for $E$.

\[
\frac{2 \frac{1}{2}}{140} = \frac{E}{1680} \quad \text{substituted}
\]

\[
\frac{5}{2} \cdot 1680 = 140E \quad \text{cross multiplied}
\]

\[
4200 = 140E \quad \text{simplified}
\]

\[
30 = E \quad \text{divided both sides by 140}
\]

Jenny needs 30 eggs to make 1680 cookies.

**Example 75.2**

It takes 3 tons of fertilizer to fertilize 170 acres. Farmer Brown wants to fertilize 1870 acres. How many tons of fertilizer does Farmer Brown need?

**Solution**

This problem concerns ratios of tons and acres. We decide to put tons in the numerator.

\[
\frac{T}{A} = \frac{T}{A}
\]

The first sentence in the problem gives us the implied ratio. It says that the ratio of tons to acres is 3 to 170. We substitute these numbers on the left. On the right we substitute 1870 for $A$. Then we solve for $T$.

\[
\frac{3}{170} = \frac{T}{1870} \quad \text{substituted}
\]

\[
3 \cdot 1870 = 170T \quad \text{cross multiplied}
\]

\[
5610 = 170T \quad \text{simplified}
\]

\[
33 = T \quad \text{divided both sides by 170}
\]

Farmer Brown needs 33 tons of fertilizer to fertilize 1870 acres.

**Practice**

a. The baker found that it took 4 huge measures of sugar to make 13 confections. The baker needed to make 143 confections. How many huge measures of sugar were needed?

**Problem Set 75**

1. The receipts for the day totaled $5200. This was only three fifths of the money needed to pay the bills. How much money was needed to pay the bills?

2. Edsel traveled the 350 miles from the factory to the River Rouge plant in 7 hours. The next day he was in a hurry to return. If he must complete the trip back to the factory in 5 hours, what should be his speed?

3. The ratio of believers to doubters at the meeting was 8 to 3. If 2400 of those in attendance were believers, how many doubters were present?

4. Four fifths of the bees were not in the hive. If 1350 bees were in the hive, how many were not in the hive?

5. Corey needs to develop 5 rolls of film. The machine takes 30 seconds to develop 2 rolls of film. How long will it take Corey to develop the 5 rolls?

6. Jonathon uses 7 sticks of vine charcoal for every 3 drawings he creates. How many drawings can he create with 23 sticks?

7. Twenty-five percent of the apartment complexes in Norman allow pets. If 85 apartment complexes allow pets, how many apartment complexes are there in Norman? Draw a diagram to help solve the problem.
Algebra ½, Lesson 75

Sample taken from Algebra ½ (Third Edition), page 239

8. Sixty-five is what percent of 325?

Graph on a number line:

9. \( x < -34 \)

10. \( x \geq 21 \)


<table>
<thead>
<tr>
<th>FRACTION</th>
<th>DECIMAL</th>
<th>PERCENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0.71</td>
<td>(b)</td>
</tr>
</tbody>
</table>

12. Write 26,900,000,000 in scientific notation.

13. What decimal part of 790 is 474?

Use the following figures for problems 14 and 15. Dimensions are in yards.

14. Find (a) the perimeter and (b) the area of the figure on the left.

15. What is the volume in cubic yards of a right solid whose base is the figure shown on the left and whose sides are 4 feet tall? Round to two decimal places.

16. Find the surface area of a right circular cylinder whose radius is 4 centimeters and whose height is 9 centimeters.

17. Use two unit multipliers to convert 1,000,000 inches to miles. Round any decimals to two places.

Simplify:

18. \( \left( \frac{2}{3} \right)^3 \)

19. \( \sqrt[3]{\frac{8}{27}} \)

20. \( \left( \frac{1}{5} \right)^3 \)

21. \( \sqrt[3]{\frac{81}{256}} \)

22. \( 10 + 3\left(3^2 + 2^2\right)(3^2 + 1) \)

23. \( 8 + -9 + 6 + -14 \)

24. \( -30 + 14 + -1 + 17 \)

25. \( \frac{11}{8} - \frac{5}{2} - \frac{1}{3} + \frac{1}{4} \)

26. \( \frac{1}{4}\left(\frac{2}{3} - \frac{1}{4}\right) + \frac{6}{7} \)

27. \( 657.12 \)

\( 0.0012 \)
Algebra ½, Lesson 75
Sample taken from Algebra ½ (Third Edition), page 240

Solve:

28. \[ \frac{3}{2} \cdot \frac{1}{2} = \frac{10}{m} \]  
29. \[ \frac{1}{4} - \frac{1}{2} = \frac{4}{12} \]

30. Which is a better deal: 3 dingoes for $57 or 5 dingoes for $99.50?
LESSON 105  Evaluating Powers of Negative Bases

Let’s examine the following expressions:

(a) \( x^2 \) and (b) \(-x^2\)

In expression (a) \( x \) is squared. If we substitute \(-2\) for \( x \), we must square \(-2\). In algebra this means

\[ (-2)^2 \]

Notice that we inserted parentheses in this expression to protect the negative sign. Thus, expression (a) tells us to multiply \( x \) by \( x \). If \( x = -2 \),

\[ x^2 = (-2)(-2) = 4 \]

Expression (b) tells us to find the opposite of the value of \( x^2 \). If we substitute \(-2\) for \( x \) in (b), we need to find the opposite of \((-2)^2\).

\[ (-(-2))^2 = -(-2)(-2) = -4 \]

There is an easy way to remember if the minus sign is included in the base of the exponent. If the symbol \(-\) is not protected by parentheses, cover it with a finger. To simplify

\[ (-2)^4 \]

we cover the minus sign with a finger.

\[ 2^4 \]

The value of \( 2^4 \) is 16. Now we remove our finger and uncover the minus sign and see that the final result is negative.

\[ -16 \]

example 105.1  Evaluate: (a) \((-3)^2\)  (b) \(-3^2\)

solution

If we try to cover the minus sign with a finger, we find that the minus sign is protected by the parentheses in (a) but not in (b).

(a) \[ (-3)^2 \]

(b) \[ -3^2 \]

So \((-3)^2\) means \((-3)(-3)\), and \(-3^2\) means \(-(3)(3)\).

(a) \((-3)^2 = 9\)  (b) \(-3^3 = -9\)

example 105.2  Evaluate: (a) \(a^2\)  (b) \(-a^2\) if \( a = -4 \)

solution

(a) We first write parentheses where we will insert the value for \( a \).

\[ (\quad)^2 \]

Then we write \(-4\) inside the parentheses and simplify.

\[ (-4)^2 = 16 \]

(b) This time we begin by writing

\[ (\quad)^2 \]

Then we write \(-4\) inside the parentheses and simplify.

\[ -(\quad)^2 \]

\[ -(-4)^2 = -16 \]
example 105.3  Evaluate: \( a^2 - ab^2 - b^2 \) if \( a = -2 \) and \( b = -3 \)

**solution**  Parentheses are not absolutely necessary, but we will use them to help us with the second term. We substitute and get

\[
(-2)^2 - (-2)(-3)^2 - (-3)^2
\]

We simplify this and get

\[
4 - (-2)(-3)^2 - 9 \quad \text{simplified}
\]

\[
4 - (-18) - 9 \quad \text{multiplied}
\]

\[
4 + 18 - 9 \quad \text{simplified}
\]

\[
13 \quad \text{added}
\]

**practice**  Evaluate:

a. \( a^2b - b \) if \( a = -2 \) and \( b = -3 \)

b. \( b^2 - a^2b \) if \( a = -2 \) and \( b = -3 \)

**problem set 105**

1. Six thirteenths of the audiophiles could discriminate between the two products. If 28 of the audiophiles could not discriminate, how many could discriminate?

2. Six sevenths of the party delegates were proclaiming their positions. If 1100 were not involved in this behavior, how many delegates were there in all?

3. The ratio of the contumacious to the affable was 2 to 17. If there were 7600, all of whom were either contumacious or affable, how many were contumacious and how many were affable?

4. Lori studied fruit fly characteristics for her genetics experiment. One in 4 had white eyes; the remainder had red eyes. There were 848 flies in this generation. How many flies had red eyes?

5. Seven times a number is 9 less than the product of the number and -20. What is the number?

6. Gawain and Lancelot drove their horses through the pounding rain and covered 16 miles in 2 hours. When the rain stopped, they slowed to half that pace. How long did it take them to travel the remaining 20 miles?

7. Twenty percent chose Plain Vanilla and five percent chose Bubble Gum Delight. If twenty-eight selected Plain Vanilla, how many fancied Bubble Gum Delight?

8. If 90 is increased by 80 percent, what is the resulting number?

9. What percent of 60 is 78?

10. Find the volume of the right solid whose base is shown on the left and whose height is 2 meters. Dimensions are in meters.
11. Find the lateral surface area of the solid in problem 10.

12. Between which two consecutive integers is $\frac{4}{95}$?

13. Find the measures of $\angle A$ and $\angle B$, and classify $\triangle ABC$ by its sides and by its angles.

Use the distributive property to multiply:

14. $3px(p + x + 2px)$

15. $mx^2(m + mny + 3m^2)$

Simplify by adding like terms:

16. $m^2n^3 + 3mm^2m - mnm^2 + 2n^3nn$

17. $ap^3 + pap^2 - 5p^2ap$

18. What is the value of $3x + (10 - x)$ when $2x + 5 = 85$?

19. $5x - 4 = -3x + 20 + 2(x - 1)$

20. $3 - 4x - 3 = -10x + 7 + 2x + 5(1 - x)$

21. $\frac{2}{x} = \frac{11}{2}$

Simplify:

22. $[-(-6)^3] + (-3)(-4)$

23. $a^3p^2ap^2a^4aa$

24. $y^z^2xyz^4$

25. $\frac{3}{4} - 0.315$

Evaluate:

26. $m^2 + n^2m$ if $m = -8$ and $n = 4$

27. $p^3 + c^3$ if $p = -3$ and $c = -1$

28. $\frac{a^4}{b^3} - a$ if $a = 4$ and $b = -2$

29. $p^4 + \frac{9}{p}$ if $p = 4$ and $q = 2$

30. Sketch a rectangular coordinate system, and graph the line $y = \frac{1}{3}x - 1$. 

100