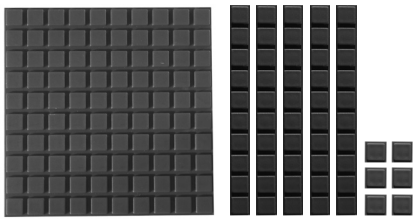
LESSON 20

Addition and Multiplication of Polynomials

Base 10 and Base X - Recall the factors of each of the pieces in base 10. The unit block (green) is 1 x 1.

The 10 bar (blue) is 1 x 10, and the 100 square (red) is 10 x 10. Each of these pieces may also be expressed in terms of exponents: 1 x 1 = 1, which is 1^0; 1 x 10 = 10, which is 10^1; 10 x 10 = 100, which is 10^2. Below is the number 156 shown with the blocks and expressed in different ways.

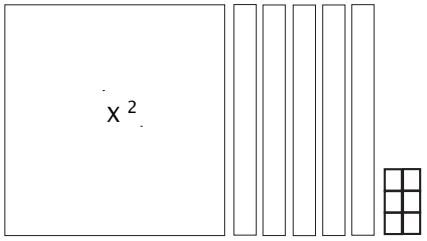


100	+	50	+	6
1 x 100	+	5 x 10	+	6 x 1
1×10^2	+	5×10^1	+	6×10^0

In the decimal system, every value is based on 10.
The decimal system is referred to as base 10.

In algebra, the unit bar is still one by one. The smooth blue piece that snaps into the back of the 10 bar is one by X, and the smooth red piece that snaps into the back of the 100 square is X by X. Each of these pieces may also be expressed in terms of exponents: $1 \cdot 1 = 1$, which is 1^0, or X^0 (which is the same thing since both are equal to one); $1 \cdot X = X$, which is X^1; and $X \cdot X = X^2$.

On the next page is the polynomial $X^2 + 5X + 6$ shown with the blocks and expressed in different ways.



$$X^2 + 5X + 6$$
$$X \cdot X + 5 \cdot X + 6 \cdot 1$$
$$1 \cdot X^2 + 5 \cdot X^1 + 6 \cdot X^0$$

In algebra, every value is based on X.
Algebra is arithmetic in base X.

Kinds of Polynomials - *Polynomial* derives from polys (many) and nomen (name), so literally it means "many names." If a polynomial has three components, it is called a *trinomial* (tri- meaning "three"). A *binomial* (bi- meaning "two") has two parts. A *monomial* (mono- meaning "one") has one part.

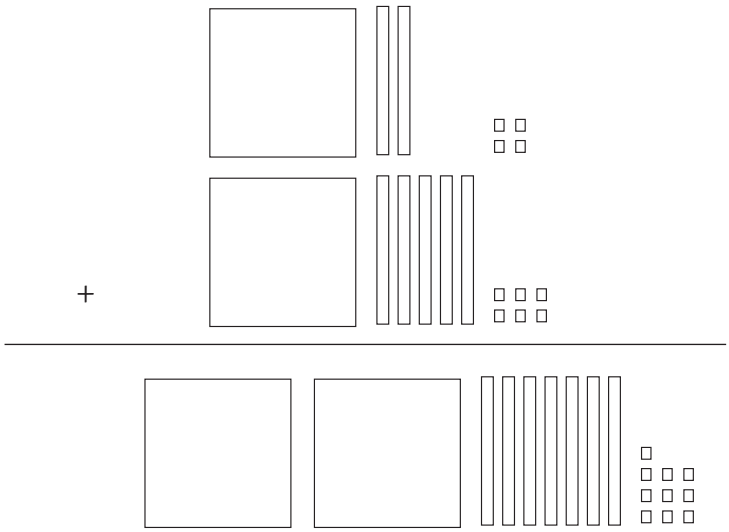
In the next few sections, whenever you feel the need to reassure yourself that you are on the right track, simply change the equation from base X to base 10, and redo the problem.

On the next two pages, operations that you are familiar with, such as addition and multiplication, will now be performed with polynomials in base X instead of base 10. Take your time and remember the connection with what you already know. Someone has said, "Algebra is not difficult, just different."

Addition of Polynomials - When adding or subtracting polynomials, remember that "to combine, they must be the same kind." Units may be added (or subtracted) with other units, Xs with Xs, X^2s with X^2s, etc. Since we don't know what the value is for X, all the addition and subtraction is done in the coefficients. Read through the following examples for clarity. The gray inserts are used to show -X.

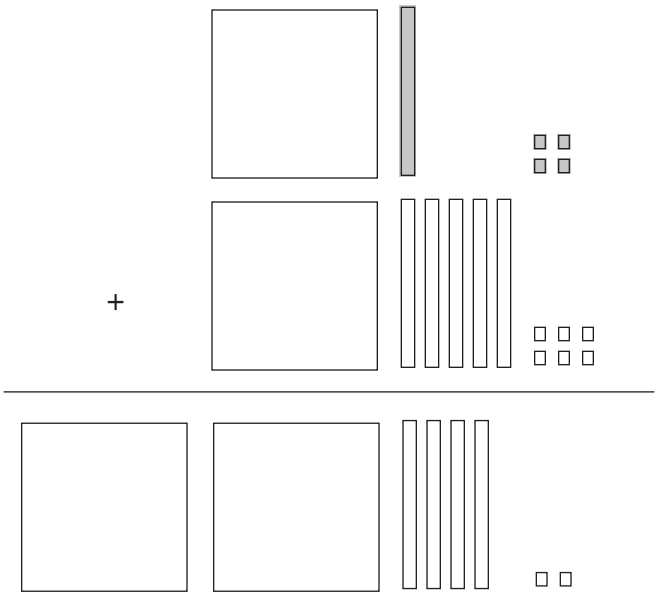
Example 1

$$\begin{array}{r} X^2 + 2X + 4 \\ +\ X^2 + 5X + 6 \\ \hline 2X^2 + 7X + 10 \end{array}$$



Example 2

$$\begin{array}{r} X^2 - X - 4 \\ +\ X^2 + 5X + 6 \\ \hline 2X^2 + 4X + 2 \end{array}$$



Multiplication of Polynomials - When we multiply two binomials, the result is a trinomial. I like to use the same format for multiplying a binomial as for multiplying any double-digit number in the decimal system.

Let's look at a problem in the decimal system using expanded notation.

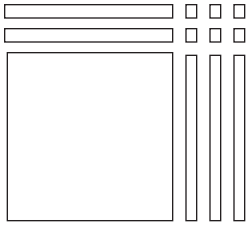
Example 3

$$\begin{array}{r} 13 \\ \times\ 12 \\ \hline \end{array} \quad = \quad \begin{array}{r} 10 + 3 \\ \times\ \ 10 + 2 \\ \hline 20 + 6 \\ 100 + 30 \\ \hline 100 + 50 + 6 \end{array}$$

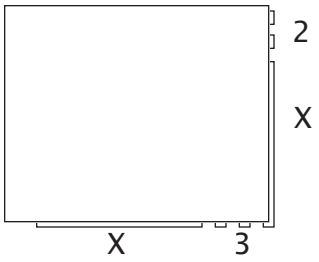
If this were in base X instead of base 10, it would look like this:

$$\begin{array}{r} X + 3 \\ \times\ \ X + 2 \\ \hline 2X + 6 \\ X^2 + 3X \\ \hline X^2 + 5X + 6 \end{array}$$

The area, or product, of this rectangle is $X^2 + 5X + 6$.
Do you see it?

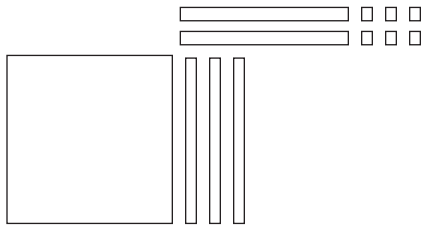


In this rectangle, we cover up most of it to reveal the factors, which are (X + 3) over and (X + 2) up.



The written equivalent of the picture looks just like double-digit multiplication, which it is.

$$\begin{array}{rr} & X + 3 \\ \times & X + 2 \\ \hline & 2X + 6 \\ X^2 + 3X & \\ \hline X^2 + 5X + 6 & \end{array}$$



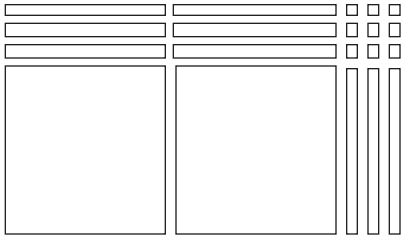
Example 4

$$\begin{array}{rr} & 23 \\ \times & 13 \\ \hline & \end{array} \quad = \quad \begin{array}{rr} & 20 + 3 \\ \times & 10 + 3 \\ \hline & 60 + 9 \\ 200 + 30 & \\ \hline 200 + 90 + 9 & \end{array}$$

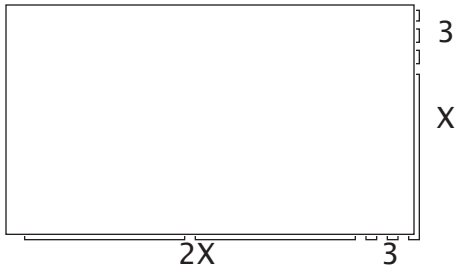
If this were in base X instead of base 10, it would look like this:

$$\begin{array}{rr} & 2X + 3 \\ \times & X + 3 \\ \hline & 6X + 9 \\ 2X^2 + 3X & \\ \hline 2X^2 + 9X + 9 & \end{array}$$

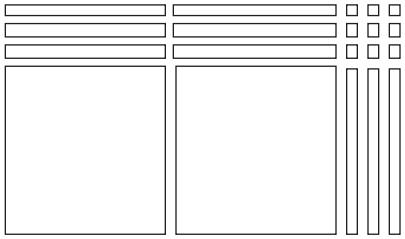
The area, or product, of this rectangle is $2X^2 + 9X + 9$. Do you see it?



In this rectangle, we cover up most of it to reveal the factors, which are (2X + 3) over and (X + 3) up.

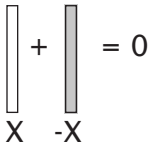


The written equivalent of the picture looks just like double-digit multiplication, which it is.



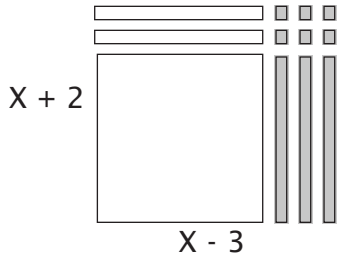
$$\begin{array}{r} 2X + 3 \\ \times \quad X + 3 \\ \hline 6X + 9 \\ 2X^2 + 3X \\ \hline 2X^2 + 9X + 9 \end{array}$$

You can also multiply binomials that include negative numbers. To show -X, use the gray inserts. The addition identity tells us that X + (-X) = 0.



Example 5

$$\begin{array}{rr} & X - 3 \\ \times & X + 2 \\ \hline & 2X - 6 \\ X^2 - 3X & \\ \hline X^2 - & X - 6 \end{array}$$



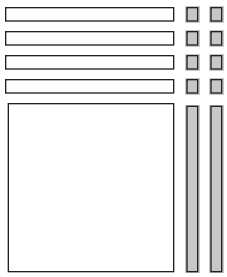
Here it is in base 10:

$$\begin{array}{rrr} & 10 - 3 & = 7 \\ \times & 10 + 2 & = 12 \\ \hline & 20 - 6 & 84 \\ 100 - 30 & & \\ \hline 100 - 10 - 6 = 84 & & \end{array}$$

Look at the next two examples carefully.

Example 6

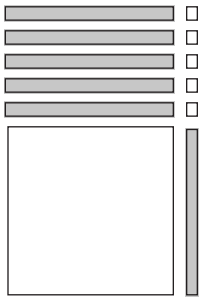
$(X - 2)(X + 4)$



$$\begin{array}{rr} & X - 2 \\ \times & X + 4 \\ \hline & 4X - 8 \\ X^2 - 2X & \\ \hline X^2 + 2X - 8 & \end{array}$$

Example 7

$(X - 1)(X - 5)$



$$\begin{array}{rr} & X - 1 \\ \times & X - 5 \\ \hline & -5X + 5 \\ X^2 - X & \\ \hline X^2 - 6X + 5 & \end{array}$$

20A

Build.

1. $X^2 + 11X + 2$

2. $X^2 + 6X + 8$

3. $X^2 - 8$

Build and add.

4. $X^2 - 6X + 3$
   $+ 3X^2 + 7X - 9$

5. $X^2 - 8$
   $+ X^2 + 6X - 7$

6. $2X^2 + 10X + 7$
   $+ 2X^2 - 8X - 9$

Build a rectangle and find the area (product).

7. $(X + 1)(X + 2) =$

8. $(X + 4)(X + 3) =$

9. $(X + 1)(X + 5) =$

Multiply.

10. $(3x + 2)(x + 1)$

11. $(5x + 5)(x + 2)$

12. $(2x + 1)(x + 5)$

13. $(x + 8)(3x + 5)$

14. $(x + 3)(2x + 1)$

15. $(3x + 2)(2x + 1)$

16. $(4x + 2)(x + 3)$

17. $(2x - 5)(x + 2)$

18. $(3x + 5)(3x - 1)$

Build.

1. $X^2 - 3X - 7$

2. $2X^2 - 7X - 3$

3. $X^2 + 5X + 9$

Build and add.

4. $X^2 + 3X + 2$
$+ X^2 + 7X + 12$

5. $X^2 + 6X + 5$
$+ 3X^2 - X - 2$

6. $5X^2 - 5X - 10$
$+ 2X^2 + 11X + 5$

Build a rectangle and find the area (product).

7. $(X + 4)(X + 5) =$

8. $(X + 7)(X + 3) =$

9. $(X + 4)(X + 8) =$

Multiply.

10. $(7x + 1)(x + 2)$

11. $(3x + 7)(x + 6)$

12. $(2x + 8)(3x + 1)$

13. $(x + 8)(x - 3)$

14. $(2x - 1)(x + 9)$

15. $(3x + 5)(x + 2)$

16. $(4x - 2)(x - 3)$

17. $(5x + 2)(3x - 3)$

18. $(3x + 7)(4x + 2)$

Build and add.

1. $3X^2 + 7X + 6$
   $+ \ X^2 + 2X + 3$

2. $2X^2 + 5X + 1$
   $+ \quad X^2 + 3X + 4$

3. $4X^2 + 8X + 2$
   $+ \ -X^2 + 3X - 1$

Build a rectangle and find the area (product).

4. $(X + 4)(X + 8) =$

5. $(X + 5)(X + 2) =$

6. $(X + 2)(X + 6) =$

Multiply.

7. $3X + 6$
   $\times \ X + 2$

8. $2X + 5$
   $\times \ X + 3$

9. $4X - 5$
   $\times \ X + 1$

10. Write on one line: $\frac{1}{X^{-4}}$

11. Rewrite using positive exponents: X^{-3}

Simplify. Write expressions with exponents on one line.

12. $5^2 \times 3^0 \times 5^{-4} =$

13. $A^4 \div A^7 =$

14. $(5^2)^5 =$

15. $(5)^{12} = (5^3)^{?} =$

16. $\sqrt{196}$ =

17. $C^{-5} \times C^{2}$ =

18. The base of a rectangle is X + 4, and the height is X + 5. What is the area of the rectangle? (Remember that the area of a rectangle is base times the height.)

19. Find the area of the rectangle in #18 if X equals six.

20. Take two times the base and height of the rectangle in #18, using the distributive property, and then find the polynomial that expresses the new area.

Build and add.

1. $X^2 - 3X - 7$
$+ 2X^2 + 4X - 4$

2. $X^2 + 11X + 2$
$+ 3X^2 - 4X + 6$

3. $X^2 - 10X - 5$
$+ -2X^2 - X + 14$

Build a rectangle and find the area (product).

4. $(X + 2)(X + 7) =$

5. $(2X + 3)(X + 4) =$

6. $(X + 1)(X + 9) =$

Multiply.

7. $2X + 4$
$\times \quad X + 3$

8. $3X - 1$
$\times \quad X + 4$

9. $2X - 3$
$\times \quad X - 4$

10. Write on one line: $\frac{1}{X^4}$

11. Rewrite using positive exponents: $\frac{1}{Y^{-5}}$

Simplify. Write expressions with exponents on one line.

12. $3^7 \times 4^3 \times 4^{-2} =$

13. $B^5 \div B^1 =$

14. $(8^3)^6 =$

15. $(2)^{15} = (2^3)^? =$

16. $\sqrt{225} =$

17. $D^{-3} \times D^8 \times D^{-7} =$

18. The base of a rectangle is 2X + 4, and the height is X + 4. What is the area of the rectangle?

19. Find the area of the rectangle in #18 if X equals 10.

20. The area of a second rectangle is $X^2 + 3X + 1$. What is the sum of the area of the two rectangles (from #18 and #20)?

Build and add.

1. $X^2 + 3X - 2$
$+ \; X^2 + 4X + 3$

2. $3X^2 + 2X - 1$
$+ \; 2X^2 - 2X + 8$

3. $5X^2 + 4X + 7$
$+ \; -X^2 + 3X + 7$

Build a rectangle and find the area (product).

4. $(X + 3)(X + 3) =$

5. $(2X + 4)(X + 2) =$

6. $(3X)(X + 2) =$

Multiply.

7. $2X - 3$
$\times \; X - 2$

8. $X - 1$
$\times \; X - 6$

9. $2X + 2$
$\times \; X - 3$

10. Write on one line: $\frac{1}{X^5}$

11. Rewrite using positive exponents: Y^{-2}

Simplify. Write expressions with exponents on one line.

12. $7^{-2} \times 7^5 \div 7^{-2} =$

13. $A^7 \div B^3 =$

Simplify. Write expressions with exponents on one line.

14. $(5^2)^5 =$

15. $(5)^{12} = (5^3)^? =$

16. $-\sqrt{169} =$

17. $C^0 C^{-4} D^8 D^{-7} D^{-3} C^3 =$

18. Stephanie's savings are represented by 3N + 4, and Chuck's are represented by 2N + 5. Write an expression representing their combined savings.

19. Stephanie and Chuck have each been saving as described in #18 for 10 weeks (N), what is the total amount they have saved?

20. The base of a rectangle is 2Y + 7, and the height is 7Y + 5. What is the area of the rectangle?

Here are some more problems involving exponents.

Follow the directions and answer the questions.

1. Suppose that m represents the mass in grams of a substance that halves in size each month. You can find the value for each month simply by dividing the value for the previous value by two.

x (number of months)	0	1	2	3	4	
m (mass in grams)	200					

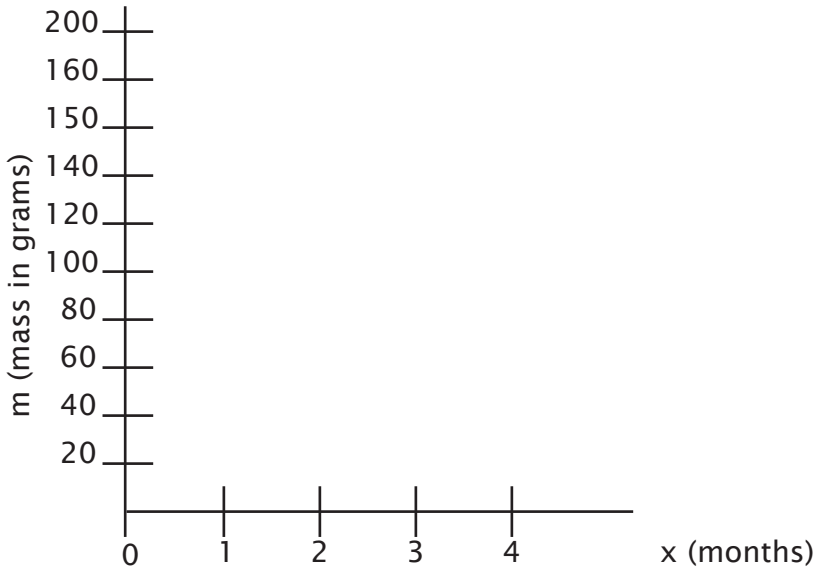
2. What was the mass of the substance when measuring began? (time = 0)

3. How long will it be until there are 100 grams remaining?

4. How long will it be until there are only 50 grams remaining?

5. What is the mass of the substance after four months?

6. Make a graph showing the first five months of decrease of the substance described on the previous page.



In real life, a scientist may wish to find the value of m for a certain number of months without finding every value in between. In this case, $m = 200(.5)^x$, where x stands for the number of months. Compare the example to the corresponding value on your chart.

Example $m = 200(.5)^x$. Find the value of m after four months.
$m = 200(.5)^4$ $m = 200(.0625) = 12.5$ grams

7. Use the equation given above to find the mass of the substance after six months.

1. $X^2 + 2X + 2$ is a:

I. polynomial II. trinomial
III. binomial IV. monomial

A. I and II
B. I and IV
C. I only
D. II only
E. III only

2. $X^2 + 3X + 2$
$+ X^2 + 4X + 5$

A. $X^2 + 7X + 7$
B. $2X^2 + 7X + 3$
C. $9X + 7$
D. $2X^2 + 7X + 7$
E. $2X^2 - X + 7$

3. $X^2 + X + 10$
$+ X^2 - 2X + 4$

A. $2X^2 - X + 14$
B. $X^2 - X + 14$
C. $-X + 6$
D. $2X^2 - 3X - 6$
E. $2X^2 + X + 14$

4. $X^2 + 8X + 6$
$+ X^2 - 3X - 1$

A. $X^2 + 5X + 5$
B. $2X^2 - 5X - 5$
C. $-11X + 7$
D. $2X^2 + 11X + 7$
E. $2X^2 + 5X + 5$

5. $X^2 - 5X - 2$
$+ X^2 - 4X - 3$

A. $X^2 + 9X + 5$
B. $9X + 5$
C. $2X^2 - X - 1$
D. $X^2 - 9X - 5$
E. $2X^2 - 9X - 5$

6. What is the sum of $2X + 3$ and $4X - 5$?

A. $6X^2 - 2$
B. $6X + 2$
C. $6X - 2$
D. $6X + 8$
E. $2X + 2$

7. What is the sum of $2X^2 - 9X + 5$ and $X^2 + 4X - 1$?

A. $3X^2 + 5X + 4$
B. $3X^2 - 5X + 4$
C. $X^2 - 5X + 4$
D. $3X^2 + 13X + 4$
E. $3X^2 - 5X + 6$

8. $4X + 3$
$\times X + 1$

A. $5X^2 + 5X + 4$
B. $11X + 3$
C. $4X^2 + 7X + 3$
D. $4X^2 + 7X + 4$
E. $4X^2 + X + 3$

9. $X + 3$
$\times\ X + 2$

A. $X^2 + 6X + 5$
B. $X^2 + 5X + 6$
C. $2X^2 + 5X + 6$
D. $X^2 + X + 5$
E. $X^2 + X + 6$

10. The product of $X + 4$ and $X - 2$ is:
A. $X^2 + 2X - 8$
B. $X^2 - 2X - 8$
C. $2X^2 + 6X - 8$
D. $X^2 - 6X - 8$
E. $X^2 - 2X + 8$

11. Multiply $X + 1$ and $X + 5$.
A. $X^2 + 5X + 6$
B. $X^2 + 6X - 5$
C. $X^2 + 6X + 5$
D. $X^2 + 5X + 4$
E. $2X^2 + 6X + 5$

12. Multiply $X - 3$ and $X - 6$.
A. $X^2 + 9X - 18$
B. $X^2 + 9X + 18$
C. $2X^2 - 9X + 18$
D. $X^2 - 9X + 18$
E. $X^2 - 18X - 9$

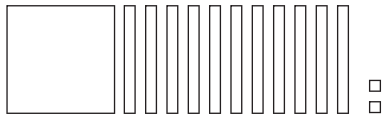
13. If $7X + 1$ and $X + 2$ are multiplied, the first term of the answer would be:
A. X^2
B. $7X^2$
C. $14X^2$
D. $2X^2$
E. $7X$

14. If $2X + 4$ and $X + 5$ are multiplied, the first term of the answer would be:
A. $3X^2$
B. $2X^2$
C. $10X^2$
D. $8X^2$
E. $20X^2$

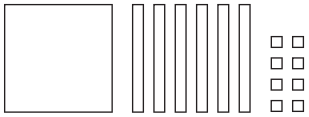
15. When we multiply 2 binomials, the result is a(n):
A. binomial
B. trinomial
C. monomial
D. integer
E. inequality

Lesson Practice 20A

1. $X^2 + 11X + 2$



2. $X^2 + 6X + 8$

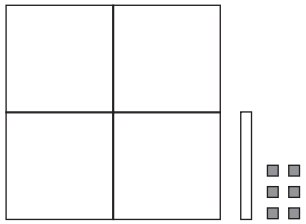


3. $X^2 - 8$

4. $X^2 - 6X + 3$

$3X^2 + 7X - 9$

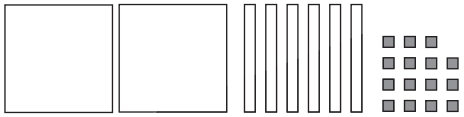
$4X^2 + X - 6$



5. $X^2 - 8$

$X^2 + 6X - 7$

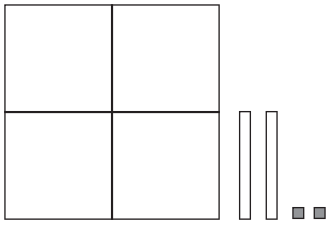
$2X^2 + 6X - 15$



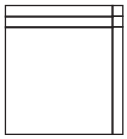
6. $2X^2 + 10X + 7$

$2X^2 - 8X - 9$

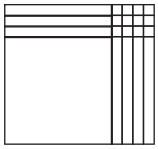
$4X^2 + 2X - 2$



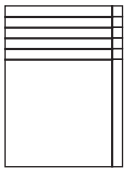
7. $(X+1)(X+2) = X^2 + 3X + 2$



8. $(X+4)(X+3) = X^2 + 7X + 12$



9. $(X+1)(X+5) = X^2 + 6X + 5$



10.

$3X + 2$

$\times X + 1$

$3X + 2$

$3X^2 + 2X$

$3X^2 + 5X + 2$

11.

$5X + 5$

$\times X + 2$

$10X + 10$

$5X^2 + 5X$

$5X^2 + 15X + 10$

12.

$2X + 1$

$\times X + 5$

$10X + 5$

$2X^2 + X$

$2X^2 + 11X + 5$

13.

$X + 8$

$\times 3X + 5$

$5X + 40$

$3X^2 + 24X$

$3X^2 + 29X + 40$