RIGHTSTART™ MATHEMATICS

by Joan A. Cotter, Ph.D.
with Tracy Mittleider, MSEd

LEVEL B LESSONS

Second Edition

Activities for Learning, Inc.

A special thank you to Kathleen Cotter Lawler for all her work on the preparation of this manual.

Note: Rather than use the designations, Kindergarten, First Grade, ect., to indicate a grade, levels are used. Level A is kindergarten, Level B is first grade, and so forth.

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www.RightStartMath.com

For more information: info@RightStartMath.com
Supplies may be ordered from: www.RightStartMath.com

Activities for Learning, Inc.
PO Box 468, 321 Hill Street
Hazelton, ND 58544-0468
United States of America
888-775-6284 or 701-782-2000
701-782-2007 fax

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How This Program Was Developed

We have been hearing for years that Japanese students do better than U.S. students in math in Japan. The Asian students are ahead by the middle of first grade. And the gap widens every year thereafter.

Many explanations have been given, including less diversity and a longer school year. Japanese students attend school 240 days a year.

A third explanation given is that the Asian public values and supports education more than we do. A first grade teacher has the same status as a university professor. If a student falls behind, the family, not the school, helps the child or hires a tutor. Students often attend after-school classes.

A fourth explanation involves the philosophy of learning. Asians and Europeans believe anyone can learn mathematics or even play the violin. It is not a matter of talent, but of good teaching and hard work.

Although these explanations are valid, I decided to take a careful look at how mathematics is taught in Japanese first grades. Japan has a national curriculum, so there is little variation among teachers.

I found some important differences. One of these is the way the Asians name their numbers. In English we count ten, eleven, twelve, thirteen, and so on, which doesn't give the child a clue about tens and ones. But in Asian languages, one counts by saying ten-1, ten-2, ten-3 for the teens, and 2-ten 1, 2-ten 2, and 2-ten 3 for the twenties.

Still another difference is their criteria for manipulatives. Americans think the more the better. Asians prefer very few, but insist that they be imaginable, that is, visualizable. That is one reason they do not use colored rods. You can imagine the one and the three, but try imagining a brown eight–the quantity eight, not the color. It cannot be done without grouping.

Another important difference is the emphasis on non-counting strategies for computation. Japanese children are discouraged from counting; rather they are taught to see quantities in groups of fives and tens.

For example, when an American child wants to know 9 + 4, most likely the child will start with 9 and count up 4. In contrast, the Asian child will think that if he takes 1 from the 4 and puts it with the 9, then he will have 10 and 3, or 13. Unfortunately, very few American first-graders at the end of the year even know that 10 + 3 is 13.

I decided to conduct research using some of these ideas in two similar first grade classrooms. The control group studied math in the traditional workbook-based manner. The other class used the lesson plans I developed. The children used that special number naming for three months.

They also used a special abacus I designed, based on fives and tens. I asked 5-year-old Stan how much is 11 + 6. Then I asked him how he knew. He replied, "I have the abacus in my mind."

The children were working with thousands by the sixth week. They figured out how to add 4-digit numbers on paper after learning how on the abacus.

Every child in the experimental class, including those enrolled in special education classes, could add numbers like 9 + 4, by changing it to 10 + 3.

I asked the children to explain what the 6 and 2 mean in the number 26. Ninety-three percent of the children in the experimental group explained it correctly while only 50% of third graders did so in another study.

I gave the children some base ten rods (none of them had seen them before) that looked like ones and tens and asked them to make 48. Then I asked them to subtract 14. The children in the control group counted 14 ones, while the experimental class removed 1 ten and 4 ones. This indicated that they saw 14 as 1 ten and 4 ones and not as 14 ones. This view of numbers is vital to understanding algorithms, or procedures, for doing arithmetic.

I asked the experimental class to mentally add 64 + 20, which only 52% of nine-year-olds on the 1986 National test did correctly; 56% of those in the experimental class could do it.

Since children often confuse columns when taught traditionally, I wrote 2304 + 86 = horizontally and asked them to find the sum any way they liked. Fifty-six percent did so correctly, including one child who did it in his head.

The following year I revised the lesson plans and both first grade classes used these methods. I am delighted to report that on a national standardized test, both classes scored at the 98th percentile.

Joan A. Cotter, Ph.D.

Some General Thoughts on Teaching Mathematics

1. Only five percent of mathematics should be learned by rote; 95 percent should be understood.
2. Real learning builds on what the child already knows. Rote teaching ignores it.
3. Contrary to the common myth, "young children can think both concretely and abstractly. Development is not a kind of inevitable unfolding in which one simply waits until a child is cognitively 'ready.'" —*Foundations for Success* NMAP
4. What is developmentally appropriate is not a simple function of age or grade, but rather is largely contingent on prior opportunities to learn." —Duschl & others
5. Understanding a new model is easier if you have made one yourself. So, a child needs to construct a graph before attempting to read a ready-made graph.
6. Good manipulatives cause confusion at first. If a new manipulative makes perfect sense at first sight, it is not needed. Trying to understand and relate it to previous knowledge is what leads to greater learning. —Richard Behr & others.
7. According to Arthur Baroody, "Teaching mathematics is essentially a process of translating mathematics into a form children can comprehend, providing experiences that enable children to discover relationships and construct meanings, and creating opportunities to develop and exercise mathematical reasoning."
8. Lauren Resnick says, "Good mathematics learners expect to be able to make sense out of rules they are taught, and they apply some energy and time to the task of making sense. By contrast, those less adept in mathematics try to memorize and apply the rules that are taught, but do not attempt to relate these rules to what they know about mathematics at a more intuitive level."
9. Mindy Holte puts learning the facts in proper perspective when she says, "In our concern about the memorization of math facts or solving problems, we must not forget that the root of mathematical study is the creation of mental pictures in the imagination and manipulating those images and relationships using the power of reason and logic." She also emphasizes the ability to imagine or visualize, an important skill in mathematics and other areas.
10. The only students who like flash cards are those who do not need them.
11. Mathematics is not a solitary pursuit. According to Richard Skemp, solitary math on paper is like reading music, rather than listening to it: "Mathematics, like music, needs to be expressed in physical actions and human interactions before its symbols can evoke the silent patterns of mathematical ideas (like musical notes), simultaneous relationships (like harmonies) and expositions or proofs (like melodies)."
12. "More than most other school subjects, mathematics offers special opportunities for children to learn the power of thought as distinct from the power of authority. This is a very important lesson to learn, an essential step in the emergence of independent thinking." —*Everybody Counts*

13. The role of the teacher is to encourage thinking by asking questions, not giving answers. Once you give an answer, thinking usually stops.
14. Putting thoughts into words helps the learning process.
15. Help the children realize that it is their responsibility to ask questions when they do not understand. Do not settle for "I don't get it."
16. The difference between a novice and an expert is that an expert catches errors much more quickly. A violinist adjusts pitch so quickly that the audience does not hear it.
17. Europeans and Asians believe learning occurs not because of ability, but primarily because of effort. In the ability model of learning, errors are a sign of failure. In the effort model, errors are natural. In Japanese classrooms, the teachers discuss errors with the whole class.
18. For teaching vocabulary, be sure either the word or the concept is known. For example, if a child is familiar with six-sided figures, we can give him the word, hexagon. Or, if he has heard the word, multiply, we can tell him what it means. It is difficult to learn a new concept and the term simultaneously.
19. Introduce new concepts globally before details. This lets the children know where they are headed.
20. Informal mathematics should precede paper and pencil work. Long before a child learns how to add fractions with unlike denominators, she should be able to add one half and one fourth mentally.
21. Some pairs of concepts are easier to remember if one of them is thought of as dominant. Then the non-dominant concept is simply the other one. For example, if even is dominant over odd; an odd number is one that is not even.
22. Worksheets should also make the child think. Therefore, they should not be a large collection of similar exercises, but should present a variety. In RightStart™ Mathematics, they are designed to be done independently.
23. Keep math time enjoyable. We store our emotional state along with what we have learned. A person who dislikes math will avoid it and a child under stress stops learning. If a lesson is too hard, stop and play a game. Try the lesson again later.
24. In Japan students spend more time on fewer problems. Teachers do not concern themselves with attention spans as is done in the U.S.
25. In Japan the goal of the math lesson is that the student has understood a concept, not necessarily has done something (a worksheet).
26. The calendar must show the entire month, so the children can plan ahead. The days passed can be crossed out or the current day circled.
27. A real mathematical problem is one in which the procedures to find the answer is not obvious. It is like a puzzle, needing trial and error. Emphasize the satisfaction of solving problems and like puzzles, of not giving away the solution to others.

RightStart™ Mathematics

Ten major characteristics make this research-based program effective:

1. Refers to quantities of up to 5 as a group; discourages counting individually. Uses fingers and tally sticks to show quantities up to 10; teaches quantities 6 to 10 as 5 plus a quantity, for example 6 = 5 + 1.
2. Avoids counting procedures for finding sums and remainders. Teaches five- and ten-based strategies for the facts that are both visual and visualizable.
3. Employs games, not flash cards, for practice.
4. Once quantities 1 to 10 are known, proceeds to 10 as a unit. Temporarily uses the "math way" of naming numbers; for example, "1 ten-1" (or "ten-1") for eleven, "1-ten 2" for twelve, "2-ten" for twenty, and "2-ten 5" for twenty-five.
5. Uses expanded notation (overlapping) place-value cards for recording tens and ones; the ones card is placed on the zero of the tens card. Encourages a child to read numbers starting at the left and not backward by starting at the ones.
6. Proceeds rapidly to hundreds and thousands using manipulatives and place-value cards. Provides opportunities for trading between ones and tens, tens and hundreds, and hundreds and thousands with manipulatives.
7. Teaches mental computation. Investigates informal solutions, often through story problems, before learning procedures.
8. Teaches four-digit addition on the abacus, letting the child discover the paper and pencil algorithm.
9. Introduces fractions with a linear visual model, including all fractions from 1⁄2 to 1⁄10. "Pies" are not used initially because they cannot show fractions greater than 1. Later, the tenths will become the basis for decimals.
10. Teaches short division (where only the answer is written down) for single-digit divisors, before long division.

Second Edition

Many changes have occurred since the first RightStart™ lessons were begun in 1994. First, mathematics is used more widely in many fields, for example, architecture, science, technology, and medicine. Today, many careers require math beyond basic arithmetic. Second, research has given us new insights into how children learn mathematics. Third, kindergarten has become much more academic, and fourth, most children are tested to ensure their preparedness for the next step.

This second edition is updated to reflect new research and applications. Topics within a grade level are always taught with the most appropriate method using the best approach with the child and teacher in mind.

Daily Lessons

Objectives. The objectives outline the purpose and goal of the lesson. Some possibilities are to introduce, to build, to learn a term, to practice, or to review.

Materials. The Math Set of manipulatives includes the specially crafted items needed to teach RightStart™ Mathematics. Occasionally, common objects such as scissors will be needed. These items are indicated by boldface type.

Warm-up. The warm-up time is the time for quick review, memory work, and sometimes an introduction to the day's topics. The dry erase board makes an ideal slate for quick responses.

Activities. The Activities for Teaching section is the heart of the lesson; it starts on the left page and continues to the right page. These are the instructions for teaching the lesson. The expected answers from the child are given in square brackets.

Establish with the children some indication when you want a quick response and when you want a more thoughtful response. Research shows that the quiet time for thoughtful response should be about three seconds. Avoid talking during this quiet time; resist the temptation to rephrase the question. This quiet time gives the slower child time to think and the quicker child time to think more deeply.

Encourage the child to develop persistence and perseverance. Avoid giving hints or explanations too quickly. Children tend to stop thinking once they hear the answer.

Explanations. Special background notes for the teacher are given in Explanations.

Worksheets. The worksheets are designed to give the children a chance to think about and to practice the day's lesson. The children are to do them independently. Some lessons, especially in the early levels, have no worksheet.

Games. Games, not worksheets or flash cards, provide practice. The games, found in the *Math Card Games* book, can be played as many times as necessary until proficiency or memorization takes place. They are as important to learning math as books are to reading. The *Math Card Games* book also includes extra games for the child needing more help, and some more challenging games for the advanced child.

In conclusion. Each lesson ends with a short summary called, "In conclusion," where the child answers a few short questions based on the day's learning.

Number of lessons. Generally, each lesson is be done in one day and each manual, in one school year. Complete each manual before going on to the next level. Other than Level A, the first lesson in each level is an introductory test with references to review lessons if needed.

Comments. We really want to hear how this program is working. Please let us know any improvements and suggestions that you may have.

Joan A. Cotter, Ph.D.
info@RightStartMath.com
www.RightStartMath.com

Lesson 20: The Commutative Property

OBJECTIVES:

1. To understand and apply the commutative property *(a + b = b + a)*

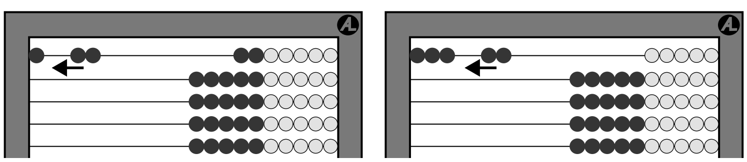
MATERIALS:

1. AL Abacus
2. Dry erase board
3. Worksheet 6, The Commutative Property

ACTIVITIES FOR TEACHING:

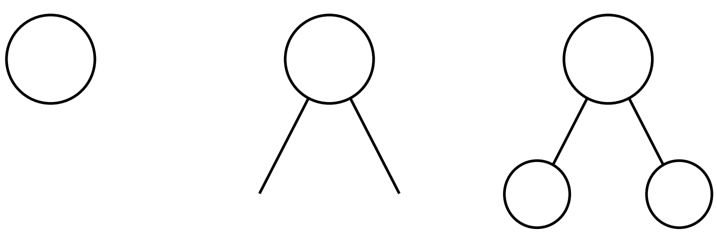
Warm-up. Ask the child to say the months of the year. Then play the Comes After game with the months. Ask: What month comes after April? [May] What month comes after August? [September] What month comes after January? [February]

Ask the child to enter 1 on her abacus and to name the quantity. [1] Ask her to add another 2 and name the amount. [3] See figure below. Continue to 9. Ask: What was special about the numbers you said? [odd numbers]



Adding 2s to count by twos.

Drawing part-whole circle sets. Show the child how to draw part-whole circle sets as shown below. First, draw the large circle. Second, draw the two lines. Third, draw the small circles by starting at the end of the lines.

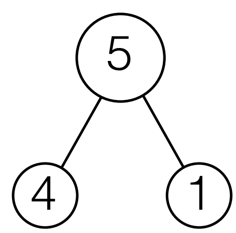


Drawing the large circle. **Drawing the lines.** **Drawing the small circles.**

Commutative property with part-whole circle sets. Ask the child to draw two part-whole circle sets. Ask her to write parts 4 and 6 in one set and parts 6 and 4 in the other as shown on the top of the next page. Ask the child to find the whole for both. [10]

EXPLANATIONS:

Part-whole circle sets are a visual tool that help children understand partitioning. The whole is written in the larger circle and the parts, in the smaller circles. Research shows children using them do better in solving story problems.

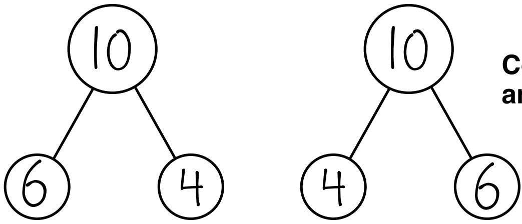


Some children discover the commutative property on their own, but others need experiences to realize and apply it.

Do not teach the term *commutative* at this point. The child must thoroughly understand the concept before the word is introduced.

ACTIVITIES FOR TEACHING:

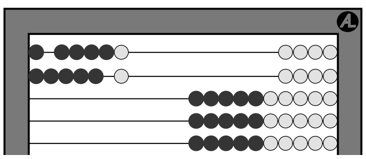
EXPLANATIONS:



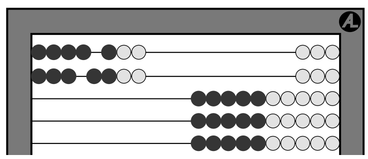
Comparing 6 + 4 and 4 + 6.

Commutative property with the abacus. Ask her to enter 5 + 1 on the first wire of her abacus and 1 + 5 on the second wire. Tell her to write the sums in the whole-circles and to write the equations. See the left figure below.

The commutative property is sometimes referred to as the commutative law. Property, meaning attribute or quality, is the preferred term.



1 + 5 = 6
5 + 1 = 6



4 + 3 = 7
3 + 4 = 7

Repeat for 4 + 3 and 3 + 4. See the right figures above. Ask her to notice how the equations are the same and how they are different. [same parts, different order] Encourage her to try her own numbers and discuss her conclusions.

Worksheet 6. This worksheet provides more practice in applying the commutative property. Using the abacus helps the child "see" the concept.

4 + 5 = **9**	7 + 2 = **9**
5 + 4 = **9**	2 + 7 = **9**
6 + 3 = **9**	3 + 5 = **8**
3 + 6 = **9**	5 + 3 = **8**
4 + 3 = **7**	7 + 1 = **8**
3 + 4 = **7**	1 + 7 = **8**
8 + 1 = **9**	3 + 7 = **10**
1 + 8 = **9**	7 + 3 = **10**

In conclusion. Write on a dry erase board 40 + 30 = 70 and 30 + 40 = 70. Ask the child: What do you notice about the equations? [The answers are the same.]

Name: ____________________ Date: ____________________

4	+	5	=	
5	+	4	=	

6	+	3	=	
3	+	6	=	

4	+	3	=	
3	+	4	=	

8	+	1	=	
1	+	8	=	

7	+	2	=	
2	+	7	=	

3	+	5	=	
5	+	3	=	

7	+	1	=	
1	+	7	=	

3	+	7	=	
7	+	3	=	

LESSON 61: ADDING SEVERAL NUMBERS

OBJECTIVES:

1. To practice adding several numbers
2. To find 2, 3, or 4 numbers that total 15

MATERIALS:

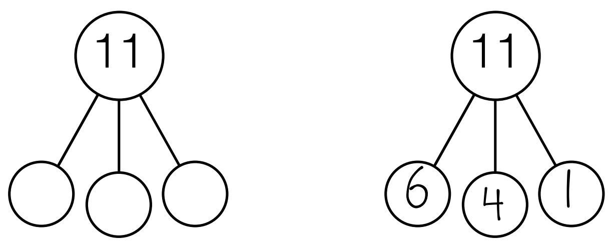
1. Dry erase board
2. Worksheet 21, Adding Several Numbers
3. Basic number cards
4. *Math Card Games* book, A53

ACTIVITIES FOR TEACHING:

Warm-up. Ask: How can you add three numbers? [First add any two numbers, then add the last number.]

Ask the child to solve the following problem using a part-whole circle set:

John has 11 apples and 3 friends to share the apples with. How could John split the apples among the 3 friends?



The part-whole circle set with three parts.

One way to partition 11 into 3 parts.

Ask the child: 9 and what equals 15? [6] 7 and what equals 15? [8] 5 and what equals 15? [10] 8 and what equals 15? [7] 6 and what equals 15? [9]

Ask: What kind of number do you always get when you add two even numbers? [even number]

Ask the child to give the ways to make 11: 3 and what? [8] 4 and what? [7] 10 and what? [1] 9 and what? [2]

EXPLANATIONS:

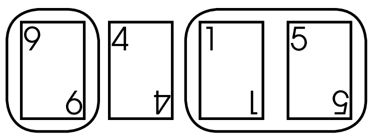
ACTIVITIES FOR TEACHING:

Worksheet 21. Give the child the worksheet. Remind her she can add the numbers in any order. The problems and solutions are below:

3 + 2 + 1 = **6**
5 + 2 + 2 = **9**
4 + 3 + 2 = **9**
1 + 2 + 7 = **10**
2 + 3 + 6 = **11**

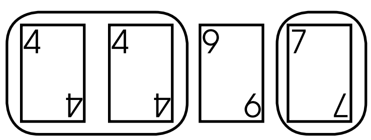
3 + 5 + 5 = **13**
2 + 7 + 8 = **17**
10 + 2 + 3 = **15**
6 + 5 + 6 = **17**
2 + 9 + 9 = **20**

Preparation for Rows and Columns game. Lay out the following basic number cards:



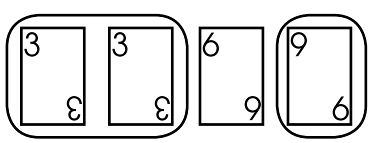
and ask the child which numbers she could use to make 15. [9, 1, 5] Ask how she found the numbers. She may see the 9 and 1 making 10 and with the 5 making 15.

Repeat for



This sum [4, 4, 7] can be seen with the 4 and 4 giving 8, which added to 7 is 15.

Repeat for



This time there are two solutions. [3, 3, 9 or 6, 9] Since the object of this new game will be to collect the most cards, the first solution is preferred.

Rows and Columns game. Play the Rows and Columns game from the *Math Card Games* book, A53.

In conclusion. Ask: What is 1 + 2 + 3 + 4 + 5? [15]

EXPLANATIONS:

There are many different ways to find the numbers. Encourage the child to discuss which ways are easiest, or fastest.

Name: ______________________________

Date: ______________________________

$3 + 2 + 1 =$ ___
$5 + 2 + 2 =$ ___
$4 + 3 + 2 =$ ___
$1 + 2 + 7 =$ ___
$2 + 3 + 6 =$ ___

$3 + 5 + 5 =$ ___
$2 + 7 + 8 =$ ___
$10 + 2 + 3 =$ ___
$6 + 5 + 6 =$ ___
$2 + 9 + 9 =$ ___

Lesson 93: Finding the Difference

OBJECTIVES:

1. To learn the term *difference*
2. To solve compare problems

MATERIALS:

1. Sums Practice 4
2. Geared clock
3. AL Abacus
4. *Math Card Games* book, S13

ACTIVITIES FOR TEACHING:

Warm-up. Ask the child to do the next two problems on Sums Practice 4 without his abacus:

$$\begin{array}{r} 1398 \\ \underline{+\ 1406} \\ \mathbf{2804} \end{array} \qquad \begin{array}{r} 3149 \\ \underline{+\ 7788} \\ \mathbf{10937} \end{array}$$

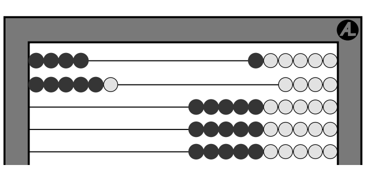
Ask: How could you use the Taking Part From Ten strategy for finding 14 – 7? [Take 4 from the 4 and 3 from the ten to get 7.] How could you use this strategy for finding 17 – 7? [Take 7 from the 7 to get ten.]

Ask: How could you use the Taking All From Ten strategy for finding 12 – 7? [Take 7 from 10 and adding 3 + 2 = 5.] How could you use this strategy for finding 13 – 6? [4 + 3 = 7]

Set the hands of the geared clock to 4:15 and ask the child to say the time. [4:15] Ask him to set his clock for various times and state those times.

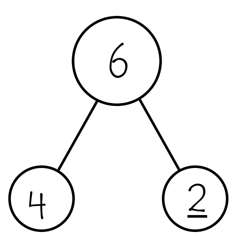
Finding differences on the abacus. Enter 4 and 6 on the top two wires of the abacus. See the left figure below. Ask the child: What is the *difference* in quantity between the 4 and 6? [2]

Ask: Did you add 4 and 6 to find the difference? [no] What did you do? [subtract] Ask him to put the numbers in a part-whole circle set. See the right figure below. Explain that the larger number goes in the whole-circle. The smaller number and difference go in the part-circles. Ask the child to write the equations.



Find the difference between 4 and 6.

$6 - 4 = \underline{2}$ or
$4 + \underline{2} = 6$

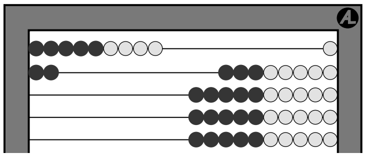


Larger number on top; smaller number and difference in part-circles.

EXPLANATIONS:

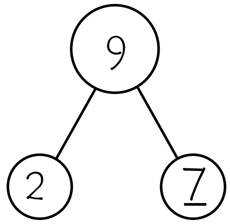
ACTIVITIES FOR TEACHING: | **EXPLANATIONS:**

Repeat for difference between 9 and 2. See figures below.



Find the difference between 9 and 2.

$9 - 2 = \underline{7}$ or
$2 + \underline{7} = 9$



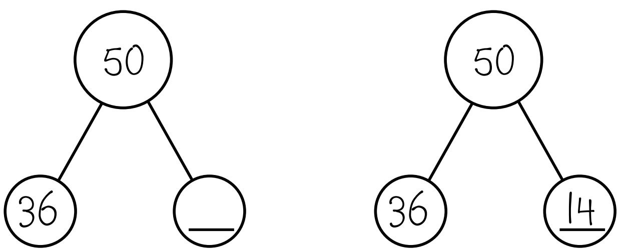
The difference is 7.

Problem. Read the following problem to the child:

> Mikayla has a book with 36 pages and Nathan has a book with 50 pages. Whose book has more pages and how many more? [Nathan, 14 more pages]

Draw a part-whole circle set and ask: Which number goes in the whole-circle? [50] What number goes in a part-circle? [36] See the left figure below. Ask: Whose book has more pages? [Nathan] How many more? [14] Ask the child to write the equation.

$$50 - 36 = \underline{14} \text{ or } 36 + \underline{14} = 50$$



The part-whole circle set for a compare problem.

Harder Difference War game. Play the Harder Difference War game from the *Math Card Games* book, S13.

A child needing an easier game could play Difference War, S12.

In conclusion. Ask the child: When you add, what do you call the answer? [sum] When you subtract, sometimes the answer is the remainder. What else can it be? [difference]

Lesson 104: Measuring with Centimeters

OBJECTIVES:

1. To measure in centimeters
2. To collect information and categorize it
3. To learn the term *data*

MATERIALS:

1. Sums Practice 6
2. Worksheet 46, Measuring with Centimeters
3. Centimeter cubes
4. One set of tangrams

ACTIVITIES FOR TEACHING:

Warm-up. Ask the child to do the last two problems on Sums Practice 6. The solutions are:

$$\begin{array}{r} 7129 \\ \underline{+\ 1516} \\ \mathbf{8645} \end{array} \qquad \begin{array}{r} 4233 \\ \underline{+\ 726} \\ \mathbf{4959} \end{array}$$

Ask: What is another word for quarter? [a fourth] What are the two names for one half of a half? [one fourth, a quarter] How many quarters in a whole? [4] How many quarters in a half? [2]

Ask: Which is more, one half or two quarters? [same] Which is less, one half or three quarters? [one half]

Ask the child to solve the following problem.

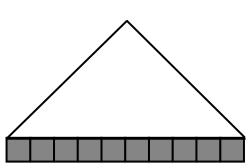
> There are 15 butterflies flying by the flowers. In the group, 6 butterflies are yellow. How many of the butterflies are not yellow? [9 butterflies]

Ask the child to mentally add 47 + 32, [77, 79] 47 + 22, [67, 69] 100 + 87, [180, 187] and 67 + 60. [127]

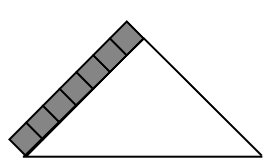
Tangram lengths. Give the child the tangrams. Ask: Are all edges of your tangram pieces the same length? [no] Explain: In this lesson you are going to find out how many different lengths the edges of the tangram pieces have. You will also find out which length is the most common and which is the least common.

Worksheet 46. Give the child the worksheet and the centimeter cubes. Show him a centimeter cube and explain that the distance along an edge is 1 centimeter.

Ask him to measure the longest side of the large triangle in centimeters. Demonstrate as shown below in the left figure. Ask: How many centimeters long is it? [10 cm]



Longest side is 10 cm.



Shorter side is 7 cm.

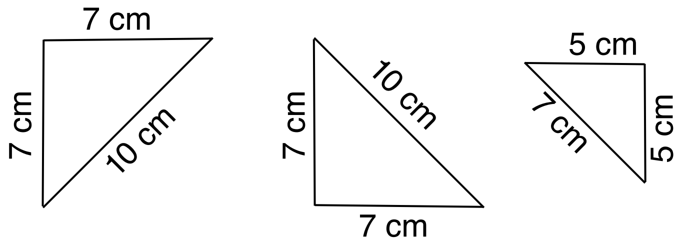
EXPLANATIONS:

According to Clements & Sarama, researchers found that children are often confused when asked to measure with various non-standard units. Only, after they are familiar with the concept of measurement, will they be able to understand the need for standard measurements.

ACTIVITIES FOR TEACHING:	EXPLANATIONS:

Next ask him to measure the side of the large triangle. [7 cm] Repeat for the other side. [7 cm] See the right figure on the previous page.

Point to the first figure from the worksheet. Ask the child what each side measured; write it on the corresponding side of the figure. Tell him that we write cm for centimeter. See the left figure below.



The lengths of the sides of the first 3 tangram pieces.

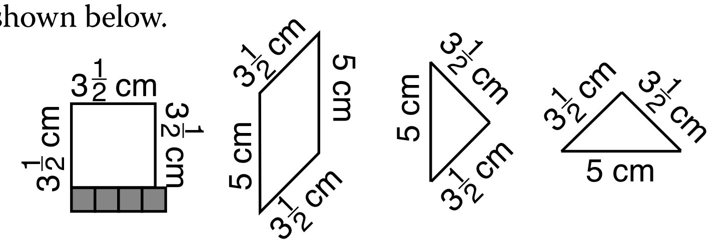
Tell the child the worksheet shows all the tangrams pieces. Tell him to measure the sides using the centimeter cubes and write the lengths for the first three triangles on their worksheet. See figures above.

Some children will realize that shapes may be identical and measuring them again is unnecessary. Other children will want to measure everything, which is necessary for them.

Measuring the square. Tell the child to measure a side of the square. Ask: Does it measure 3 cm? [too little] Does it measure 4 cm? [too much] Tell him: The side measures 3 and a part of a another centimeter. What part is it? [one half] Tell them: We say it is 3 and one half centimeters. Show them to how write $3\frac{1}{2}$ cm.

Although fractions are not common within the metric system, they are permissible.

Do the same thing with the last three pieces. Answers are shown below.



The lengths of the sides of the last 4 tangram pieces.

Worksheet Question 2. Explain to the child that he has a lot of information, called *data*; now he can organize it in the chart. First, his is to count the number of sides having 10 cm and write it below the box saying 10 cm. Next he is to find the number of sides that are 7 cm long and write it below the 7 cm. Do the same thing with the last two lengths. The solutions are:

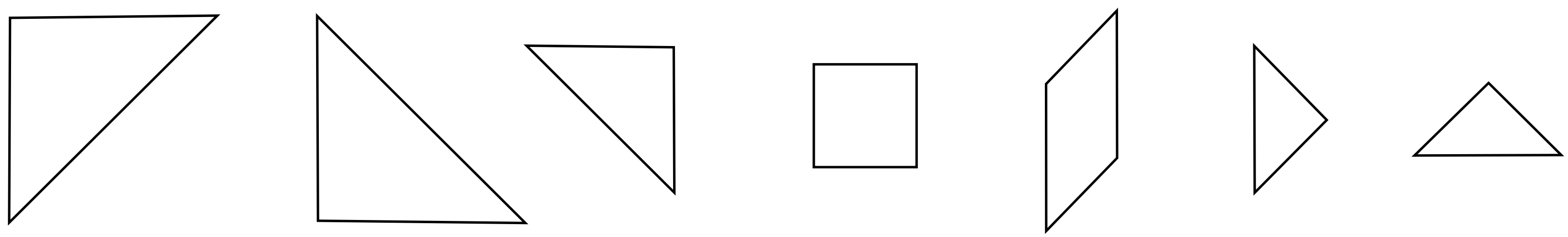
10	7	5	3
2	**5**	**6**	**$10\frac{1}{2}$**

Worksheet Question 3. Here he is to tell what he learned about the lengths.

In conclusion. Ask: Are you surprised there are only four different lengths?

Name: ______________________ Date: ______________________

1. Measure the side of each tangram piece in centimeters and write it along the edge.



2. Write the total number of sides with each measurement.

10 cm	7 cm	5 cm	$3\frac{1}{2}$ cm

3. Write about your findings.

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