RightStart™ Mathematics

by Joan A. Cotter, Ph.D.
with Kathleen Cotter Lawler

LEVEL D LESSONS

Second Edition

Activities for Learning, Inc.

A special thank you to Maren Ehley and Rebecca Walsh for their work on the final preparation of this manual.

Note: Levels are used rather than grades. For example, Level A is kindergarten and Level B is first grade and so forth.

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How This Program Was Developed

We have been hearing for years that Japanese students do better than U.S. students in math in Japan. The Asian students are ahead by the middle of first grade. And the gap widens every year thereafter.

Many explanations have been given, including less diversity and a longer school year. Japanese students attend school 240 days a year.

A third explanation given is that the Asian public values and supports education more than we do. A first grade teacher has the same status as a university professor. If a student falls behind, the family, not the school, helps the child or hires a tutor. Students often attend after-school classes.

A fourth explanation involves the philosophy of learning. Asians and Europeans believe anyone can learn mathematics or even play the violin. It is not a matter of talent, but of good teaching and hard work.

Although these explanations are valid, I decided to take a careful look at how mathematics is taught in Japanese first grades. Japan has a national curriculum, so there is little variation among teachers.

I found some important differences. One of these is the way the Asians name their numbers. In English we count ten, eleven, twelve, thirteen, and so on, which doesn't give the child a clue about tens and ones. But in Asian languages, one counts by saying ten-1, ten-2, ten-3 for the teens, and 2-ten 1, 2-ten 2, and 2-ten 3 for the twenties.

Still another difference is their criteria for manipulatives. Americans think the more the better. Asians prefer very few, but insist that they be imaginable, that is, visualizable. That is one reason they do not use colored rods. You can imagine the one and the three, but try imagining a brown eight–the quantity eight, not the color. It cannot be done without grouping.

Another important difference is the emphasis on non-counting strategies for computation. Japanese children are discouraged from counting; rather they are taught to see quantities in groups of fives and tens.

For example, when an American child wants to know 9 + 4, most likely the child will start with 9 and count up 4. In contrast, the Asian child will think that if he takes 1 from the 4 and puts it with the 9, then he will have 10 and 3, or 13. Unfortunately, very few American first-graders at the end of the year even know that 10 + 3 is 13.

I decided to conduct research using some of these ideas in two similar first grade classrooms. The control group studied math in the traditional workbook-based manner. The other class used the lesson plans I developed. The children used that special number naming for three months.

They also used a special abacus I designed, based on fives and tens. I asked 5-year-old Stan how much is 11 + 6. Then I asked him how he knew. He replied, "I have the abacus in my mind."

The children were working with thousands by the sixth week. They figured out how to add 4-digit numbers on paper after learning how on the abacus.

Every child in the experimental class, including those enrolled in special education classes, could add numbers like 9 + 4, by changing it to 10 + 3.

I asked the children to explain what the 6 and 2 mean in the number 26. Ninety-three percent of the children in the experimental group explained it correctly while only 50% of third graders did so in another study.

I gave the children some base ten rods (none of them had seen them before) that looked like ones and tens and asked them to make 48. Then I asked them to subtract 14. The children in the control group counted 14 ones, while the experimental class removed 1 ten and 4 ones. This indicated that they saw 14 as 1 ten and 4 ones and not as 14 ones. This view of numbers is vital to understanding algorithms, or procedures, for doing arithmetic.

I asked the experimental class to mentally add 64 + 20, which only 52% of nine-year-olds on the 1986 National test did correctly; 56% of those in the experimental class could do it.

Since children often confuse columns when taught traditionally, I wrote 2304 + 86 = horizontally and asked them to find the sum any way they liked. Fifty-six percent did so correctly, including one child who did it in his head.

The following year I revised the lesson plans and both first grade classes used these methods. I am delighted to report that on a national standardized test, both classes scored at the 98th percentile.

Joan A. Cotter, Ph.D.

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Some General Thoughts on Teaching Mathematics

1. Only five percent of mathematics should be learned by rote; 95 percent should be understood.
2. Real learning builds on what the child already knows. Rote teaching ignores it.
3. Contrary to the common myth, "young children can think both concretely and abstractly. Development is not a kind of inevitable unfolding in which one simply waits until a child is cognitively 'ready.'" —*Foundations for Success* NMAP
4. What is developmentally appropriate is not a simple function of age or grade, but rather is largely contingent on prior opportunities to learn." —Duschl & others
5. Understanding a new model is easier if you have made one yourself. So, a child needs to construct a graph before attempting to read a ready-made graph.
6. Good manipulatives cause confusion at first. If a new manipulative makes perfect sense at first sight, it is not needed. Trying to understand and relate it to previous knowledge is what leads to greater learning. —Richard Behr & others.
7. According to Arthur Baroody, "Teaching mathematics is essentially a process of translating mathematics into a form children can comprehend, providing experiences that enable children to discover relationships and construct meanings, and creating opportunities to develop and exercise mathematical reasoning."
8. Lauren Resnick says, "Good mathematics learners expect to be able to make sense out of rules they are taught, and they apply some energy and time to the task of making sense. By contrast, those less adept in mathematics try to memorize and apply the rules that are taught, but do not attempt to relate these rules to what they know about mathematics at a more intuitive level."
9. Mindy Holte puts learning the facts in proper perspective when she says, "In our concern about the memorization of math facts or solving problems, we must not forget that the root of mathematical study is the creation of mental pictures in the imagination and manipulating those images and relationships using the power of reason and logic." She also emphasizes the ability to imagine or visualize, an important skill in mathematics and other areas.
10. The only students who like flash cards are those who do not need them.
11. Mathematics is not a solitary pursuit. According to Richard Skemp, solitary math on paper is like reading music, rather than listening to it: "Mathematics, like music, needs to be expressed in physical actions and human interactions before its symbols can evoke the silent patterns of mathematical ideas (like musical notes), simultaneous relationships (like harmonies) and expositions or proofs (like melodies)."
12. "More than most other school subjects, mathematics offers special opportunities for children to learn the power of thought as distinct from the power of authority. This is a very important lesson to learn, an essential step in the emergence of independent thinking." —*Everybody Counts*

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13. The role of the teacher is to encourage thinking by asking questions, not giving answers. Once you give an answer, thinking usually stops.
14. Putting thoughts into words helps the learning process.
15. Help the children realize that it is their responsibility to ask questions when they do not understand. Do not settle for “I don’t get it.”
16. The difference between a novice and an expert is that an expert catches errors much more quickly. A violinist adjusts pitch so quickly that the audience does not hear it.
17. Europeans and Asians believe learning occurs not because of ability, but primarily because of effort. In the ability model of learning, errors are a sign of failure. In the effort model, errors are natural. In Japanese classrooms, the teachers discuss errors with the whole class.
18. For teaching vocabulary, be sure either the word or the concept is known. For example, if a child is familiar with six-sided figures, we can give him the word, hexagon. Or, if he has heard the word, multiply, we can tell him what it means. It is difficult to learn a new concept and the term simultaneously.
19. Introduce new concepts globally before details. This lets the children know where they are headed.
20. Informal mathematics should precede paper and pencil work. Long before a child learns how to add fractions with unlike denominators, she should be able to add one half and one fourth mentally.
21. Some pairs of concepts are easier to remember if one of them is thought of as dominant. Then the non-dominant concept is simply the other one. For example, if even is dominant over odd; an odd number is one that is not even.
22. Worksheets should also make the child think. Therefore, they should not be a large collection of similar exercises, but should present a variety. In RightStart™ Mathematics, they are designed to be done independently.
23. Keep math time enjoyable. We store our emotional state along with what we have learned. A person who dislikes math will avoid it and a child under stress stops learning. If a lesson is too hard, stop and play a game. Try the lesson again later.
24. In Japan students spend more time on fewer problems. Teachers do not concern themselves with attention spans as is done in the U.S.
25. In Japan the goal of the math lesson is that the student has understood a concept, not necessarily has done something (a worksheet).
26. The calendar must show the entire month, so the children can plan ahead. The days passed can be crossed out or the current day circled.
27. A real mathematical problem is one in which the procedures to find the answer is not obvious. It is like a puzzle, needing trial and error. Emphasize the satisfaction of solving problems and like puzzles, of not giving away the solution to others.

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RightStart™ Mathematics

Ten major characteristics make this research-based program effective:

1. Refers to quantities of up to 5 as a group; discourages counting individually. Uses fingers and tally sticks to show quantities up to 10; teaches quantities 6 to 10 as 5 plus a quantity, for example 6 = 5 + 1.
2. Avoids counting procedures for finding sums and remainders. Teaches five- and ten-based strategies for the facts that are both visual and visualizable.
3. Employs games, not flash cards, for practice.
4. Once quantities 1 to 10 are known, proceeds to 10 as a unit. Temporarily uses the "math way" of naming numbers; for example, "1 ten-1" (or "ten-1") for eleven, "1-ten 2" for twelve, "2-ten" for twenty, and "2-ten 5" for twenty-five.
5. Uses expanded notation (overlapping) place-value cards for recording tens and ones; the ones card is placed on the zero of the tens card. Encourages a child to read numbers starting at the left and not backward by starting at the ones.
6. Proceeds rapidly to hundreds and thousands using manipulatives and place-value cards. Provides opportunities for trading between ones and tens, tens and hundreds, and hundreds and thousands with manipulatives.
7. Teaches mental computation. Investigates informal solutions, often through story problems, before learning procedures.
8. Teaches four-digit addition on the abacus, letting the child discover the paper and pencil algorithm.
9. Introduces fractions with a linear visual model, including all fractions from 1⁄2 to 1⁄10. "Pies" are not used initially because they cannot show fractions greater than 1. Later, the tenths will become the basis for decimals.
10. Teaches short division (where only the answer is written down) for single-digit divisors, before long division.

Second Edition

Many changes have occurred since the first RightStart™ lessons were begun in 1994. First, mathematics is used more widely in many fields, for example, architecture, science, technology, and medicine. Today, many careers require math beyond basic arithmetic. Second, research has given us new insights into how children learn mathematics. Third, kindergarten has become much more academic, and fourth, most children are tested to ensure their preparedness for the next step.

This second edition is updated to reflect new research and applications. Topics within a grade level are always taught with the most appropriate method using the best approach with the child and teacher in mind.

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Daily Lessons

Objectives. The objectives outline the purpose and goal of the lesson. Some possibilities are to introduce, to build, to learn a term, to practice, or to review.

Materials. The Math Set of manipulatives includes the specially crafted items needed to teach RightStart™ Mathematics. Occasionally, common objects such as scissors will be needed. These items are indicated by boldface type.

Warm-up. The warm-up time is the time for quick review, memory work, and sometimes an introduction to the day's topics. The dry erase board makes an ideal slate for quick responses.

Activities. The Activities for Teaching section is the heart of the lesson; it starts on the left page and continues to the right page. These are the instructions for teaching the lesson. The expected answers from the child are given in square brackets.

Establish with the children some indication when you want a quick response and when you want a more thoughtful response. Research shows that the quiet time for thoughtful response should be about three seconds. Avoid talking during this quiet time; resist the temptation to rephrase the question. This quiet time gives the slower child time to think and the quicker child time to think more deeply.

Encourage the child to develop persistence and perseverance. Avoid giving hints or explanations too quickly. Children tend to stop thinking once they hear the answer.

Explanations. Special background notes for the teacher are given in Explanations.

Worksheets. The worksheets are designed to give the children a chance to think about and to practice the day's lesson. The children are to do them independently. Some lessons, especially in the early levels, have no worksheet.

Games. Games, not worksheets or flash cards, provide practice. The games, found in the *Math Card Games* book, can be played as many times as necessary until proficiency or memorization takes place. They are as important to learning math as books are to reading. The *Math Card Games* book also includes extra games for the child needing more help, and some more challenging games for the advanced child.

In conclusion. Each lesson ends with a short summary called, "In conclusion," where the child answers a few short questions based on the day's learning.

Number of lessons. Generally, each lesson is be done in one day and each manual, in one school year. Complete each manual before going on to the next level. Other than Level A, the first lesson in each level is an introductory test with references to review lessons if needed.

Comments. We really want to hear how this program is working. Please let us know any improvements and suggestions that you may have.

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Lesson 18: The Commutative Property

OBJECTIVES:

1. To learn the term *factor*
2. To introduce the commutative property
3. To learn the term *commutative*

MATERIALS:

1. AL Abacus
2. Dry erase board
3. *Math Card Games* book, P10

ACTIVITIES FOR TEACHING:

Warm-up. Ask: What is 4 times 1? [4] What is 8 times 1? [8] What is 4 times 2? [8] What is 8 times 2? [16] What is 4 times 3? [12] 8 times 3? [24] What is 4 times 4? [16] 8 times 4? [32] What is 4 times 5? [20] 8 times 5? [40]

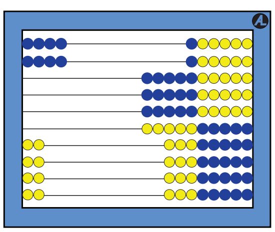
Ask: What is 4 times 6? [24] 8 times 6? [48] What is 4 times 7? [28] 8 times 7? [56] What is 4 times 8? [32] 8 times 8? [64] What is 4 times 9? [36] 8 times 9? [72] What is 4 times 10? [40] 8 times 10? [80]

EXPLANATIONS:

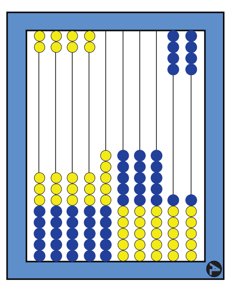
Some children may need to use the abacus for some of these warm-up questions.

The commutative property on the abacus. Give the child the abacus and dry erase board. Tell the child: Enter 4 multiplied by 2 on the top two rows of your abacus. Also enter 2 multiplied by 4 on the bottom four rows of your abacus. See the left figure below. Ask: What are the equations? [4 × 2 = 8 and 2 × 4 = 8]

The commutative property was formerly called the commutative *law*. A property is an attribute or quality.



4 × 2 and 2 × 4



4 × 2 and 2 × 4

Now tell her to turn her abacus clockwise, that is, in the same direction the hands turn on a clock. See the right figure above. Tell her to write the equations on her white board. [4 × 2 = 8 and 2 × 4 = 8]

Say: The number we multiply and the number we multiplied by are called *factors*. In the equations just written, 2 and 4 are the factors.

Tell her to enter 8 multiplied by 4 on her abacus and to write the equation. [8 × 4 = 32] Then tell her to turn her abacus clockwise and write that equation. [4 × 8 = 32] Did the order of the factors make a difference? [no]

Commutative examples. Make two columns. Label the left column “Makes a Difference” and the right side

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ACTIVITIES FOR TEACHING:

EXPLANATIONS:

"Makes No Difference." See the figure below.

Ask: Does it make any difference at a meal whether you eat beans or corn first? [no] Write it in the left column.

Ask: Does it matter if you mix the batter or bake the cake first? [yes] Write it in the right column.

Makes No Difference	Makes a Difference
Eat beans or corn	Mix the batter or bake the cake
Put on left or right shoe	Eat or peel banana

Ask: Does the order matter for peeling and eating a banana? [yes]

Ask: Do you get the same results if you first put on your left shoe or your right shoe? [yes]

Tell her to think of some examples to be recorded.

Ask: Is 89 + 3 equal to 3 + 89? [yes] Does the order make a difference in adding? [no] Write it in the left column.

Ask: In subtraction, is 5 – 3 equal to 3 – 5? [no] Put it in the right column.

Ask: For multiplication, is 5 multiplied by 2 the same as 2 multiplied by 5? [yes] Does the order make a difference in multiplying? [no] Put in the left column.

See page iii, number 18 of "Some General Thoughts on Teaching Mathematics," for additional information.

Tell her: The mathematical word for getting the same results when the order of the numbers is changed is *commutative.* Write "Commutative" above the left column and "Not commutative" above the right column as shown below.

Commutative	Not commutative
Makes No Difference	Makes a Difference
Eat beans or corn	Mix the batter or bake the cake
Put on left or right shoe	Eat or peel banana
Foot to pedal on bike	Put on shoes or socks
Mittens on hands	Dry or wash hair
89 + 3 or 3 + 89	5 – 3 or 3 – 5
2 × 5 or 5 × 2	

Multiplication Memory game. Play the Multiplication Memory game from the *Math Card Games* book, P10, using the 8s.

In conclusion. Ask: What is 8 times 3? [24] What is 3 times 8? [24] What is 8 times 7? [56] What is 7 times 8? [56] What is 9 times 8? [72]

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LESSON 29: AREA ON THE MULTIPLICATION TABLE

OBJECTIVES:

1. To review *perimeter* and *area*
2. To see area on the multiplication table
3. To introduce exponents
4. To see the symmetry of the multiplication table

MATERIALS:

1. Worksheet 15, Area on the Multiplication Table
2. Tiles
3. *Math Card Games* book, P21

ACTIVITIES FOR TEACHING:

Warm-up. Ask: How many numbers are on the addition table? [100] How many numbers are on the multiplication table? [100] What is the size of the arrays? [10 by 10] Can you use the multiplication table for adding? [no] Can you use it for multiplying? [yes]

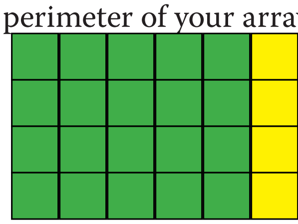
Worksheet 15. Give the child the worksheet and tiles.

Reviewing perimeter. Show a tile and say: The length of an edge of a tile is 1 inch. The distance around an object is called the *perimeter.* Ask: What is the perimeter of a tile? [4 in.]

Area. Say: How much space something takes up is called *area*. Show the tile and say: The area of a tile is 1 square inch.

Tell her to make a 6 by 4 array with the tiles as shown.

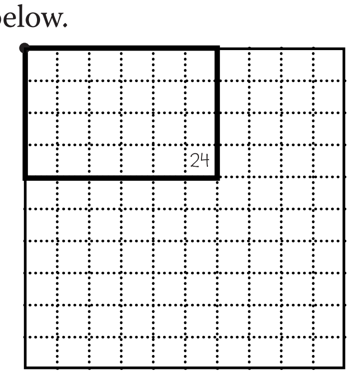
Ask: What is the perimeter of your array in inches? [20



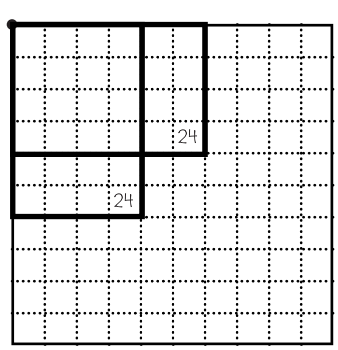
6 × 4 array

in.] What is the area of your array in square inches? [24 sq. in.] Tell the child to start at the dot on her worksheet and draw this rectangle. Tell her to write the area at the opposite corner. See the left figure below.

Tell her to repeat for a 4 × 6 array. See the right figure below.



6 × 4 array



4 × 6 array added

EXPLANATIONS:

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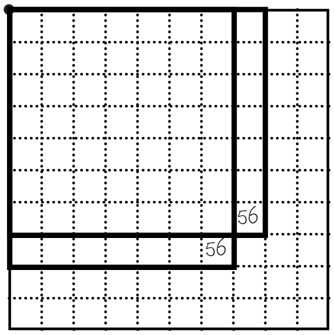
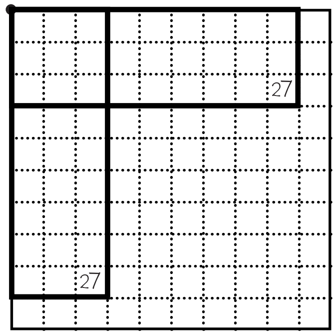
ACTIVITIES FOR TEACHING: | **EXPLANATIONS:**

Tell her to do the arrays for the second and third tables on her worksheet. The solutions are shown below.

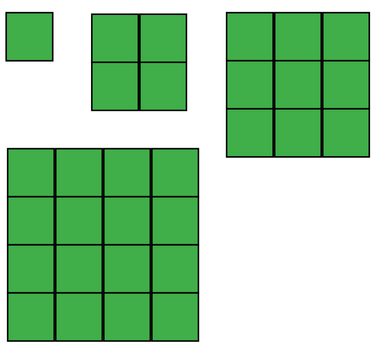


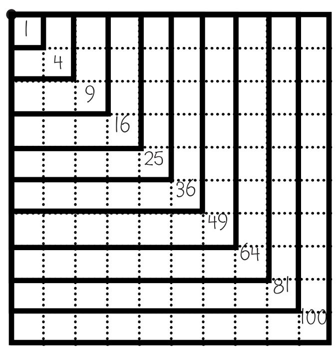
8 × 7 and 7 × 8 arrays



9 × 3 and 3 × 9 arrays

Squares on the multiplication table. For the last table on the worksheet, ask the child to construct several squares with the tiles and draw all the squares on the fourth multiplication table. See below.





The square arrays.

Writing squares with exponents. Write:

$$3 \times 3 = 3^2$$

and explain this is a shortcut for writing squares. Say: We write 3 times 3 by writing only one 3 with a little 2 after it. The little 2 means we are multiplying 3 two times. Read it as "3 squared".

Write: $5^2 =$ ___

and ask: What does this mean? [5 × 5] How much is it? [25] Repeat for 8^2 [8 × 8 = 64] and 1^2. [1 × 1 = 1]

Square Memory game. Play the Square Memory game, which is found in the *Math Card Games* book, P21. Say: You will need one card from each envelope. Take the 1-card from 1s envelope, the 4-card from the 2s envelope, and so forth up to the 100-card from the 10s envelope. Tell her to play the game twice and return the cards to the correct envelopes.

By removing these cards from the envelopes, the child may become more aware of the square numbers that are indicated on the outside of the envelopes.

In conclusion. Ask: What numbers are on the diagonal in the multiplication table? [squares] Why is 56 on the multiplication table twice? [56 is 8 × 7 and 7 × 8]

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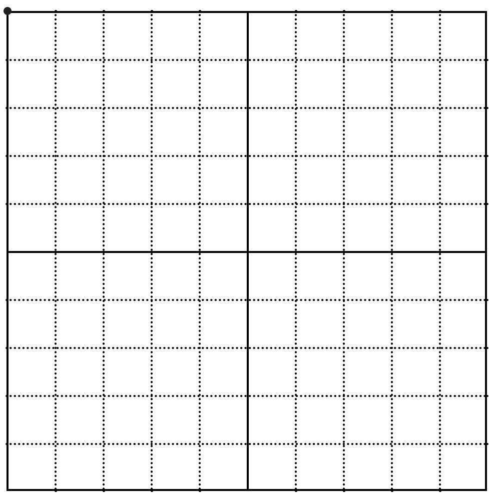
Name: ____________________________________

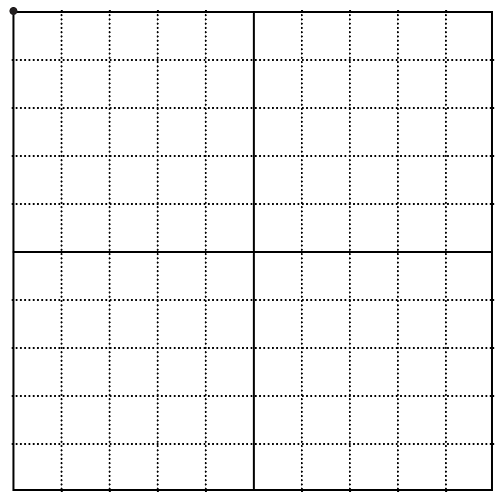
Date: ____________________________________

Start at the dot and draw rectangles for arrays. Write the area in the cell opposite the dot.

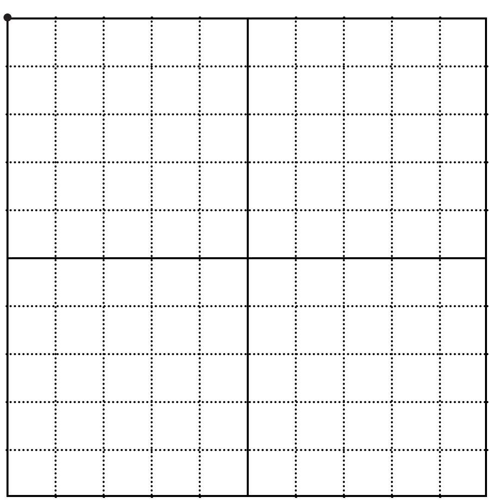
6 × 4 and 4 × 6



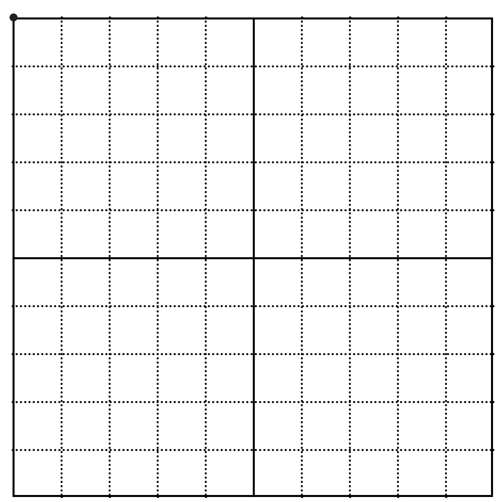
8 × 7 and 7 × 8



9 × 3 and 3 × 9



1 × 1, 2 × 2, 3 × 3, and up to 10 × 10



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Lesson 32: The Short Multiplication Table

OBJECTIVES:

1. To construct the short multiplication table
2. To use the short multiplication table

MATERIALS:

1. *Math Card Games* book, P28
2. Math journal
3. Worksheet 18, The Short Multiplication Table

ACTIVITIES FOR TEACHING:

Warm-up. Ask: What is 8 × 8? [64] What is 7 × 9? [63] What is 9 × 7? [63]

Ask: What is 7 × 7? [49] What is 8 × 6? [48] 6 × 8? [48]

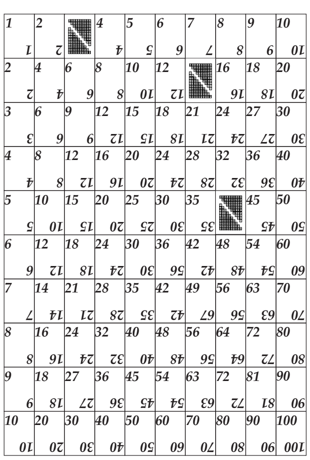
Ask: What is 6 × 6? [36] What is 7 × 5? [35] 5 × 7? [35]

Ask: What is 9 × 9? [81] What is 8 × 10? [80] 10 × 8? [80]

Weighted Multi-Fun game. Play the Weighted Multi-Fun game, found in *Math Card Games* book, P28. Tell her to write her scores in her math journal, in the same way she did for the Sum Rummy game, P3. See the example on the right. The first equation, 5 × 4, shows 5 cards played in the fourth row or column; the second equation, 4 cards in the eighth row or column. She can write several equations before summing as shown.

5 × 4 = 20
4 × 8 = 32
7 × 5 = 35
87
3 × 6 = 18
3 × 10 = 30
135

The short multiplication table. Say: There is one more activity to do with the cards at the end of the game. Tell the child to find 2 × 7 and 7 × 2. Find the duplicate products. [14] Turn face down the 14-card in the column with the higher factor. Continue with 3 × 1 and 1 × 3, also with 5 × 8 and 8 × 5. See the figure below.



The duplicate products of 7 × 2, 3 × 1, and 8 × 5 turned face down.

Tell her to repeat for all duplicate products. See figure on the next page. Tell her this is the *short multiplication table*.

EXPLANATIONS:

Maintain the card layout for the next activity.

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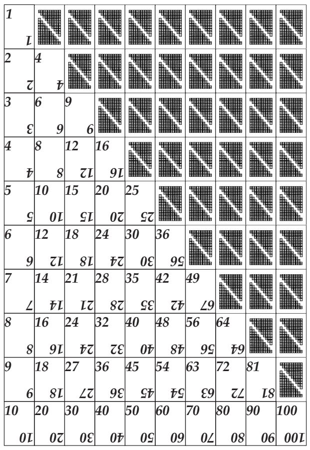
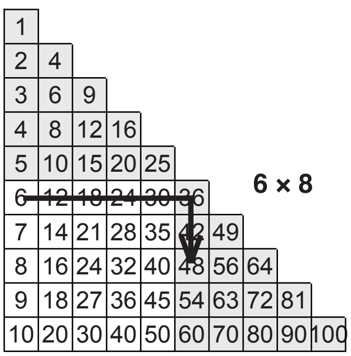
ACTIVITIES FOR TEACHING:

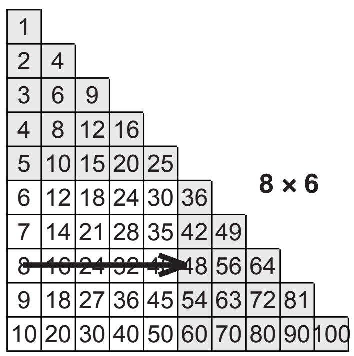


The multiplication table with the duplicates face down, resulting in the short multiplication table.

Worksheet 18. Give the child the worksheet. Tell her to find the 6 row. Say: The first six multiples are in the 6 row. Ask: Where are the rest of the 6s? [She continues down in the 36 column to 60.] See the left figure below.

Tell her to find 6 × 8. See the left figure below. Then tell her to find 8 × 6. See the right figure. Ask: What do you notice? [The product is in the same cell.]





Tell her to complete the worksheet. The solutions are below.

1									
2	4								
3	6	9							
4	8	12	16						
5	10	15	20	25					
6	12	18	24	30	36				
7	14	21	28	35	42	49			
8	16	24	32	40	48	56	64		
9	18	27	36	45	54	63	72	81	
10	20	30	40	50	60	70	80	90	100

4 × 4 = **16**
9 × 4 = **36**
8 × 7 = **56**
5 × 7 = **35**
9 × 6 = **54**
7 × 4 = **28**

4 × 5 = **20**
2 × 5 = **10**
7 × 8 = **56**
3 × 9 = **27**
6 × 9 = **54**
10 × 1 = **10**

3	8	6	4	9	2	10	7	5	9	8
× 4	× 9	× 8	× 6	× 7	× 7	× 4	× 6	× 8	× 9	× 8
12	**72**	**48**	**24**	**63**	**14**	**40**	**42**	**40**	**81**	**64**

The last number in a row **is a square number.**
Number of cells in row 7 is **7**; 8 is **8**; 5 is **5;** 10 is **10.**

In conclusion. Tell her to say the facts for the square numbers: [1 × 1 = 2, 2 × 2 = 4, . . . , 10 × 10 = 100]

EXPLANATIONS:

The short multiplication table is shown in the first printed arithmetic book, *Treviso Arithmetic*, printed in 1478.

Gather the cards that are face down into one pile and the face-up cards into another pile. Then combine the two piles so that all the cards are face up. There is no need to put the cards in the envelopes as the next games need all the cards.

No counting is necessary because the cells are grouped by fives.

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Name: ______________________________

Date: ______________________________

Short Multiplication Table

1									
2	4								
3	6	9							
4	8	12	16						
5	10	15	20	25					
6	12	18	24	30	36				
7	14	21	28	35	42	49			
8	16	24	32	40	48	56	64		
9	18	27	36	45	54	63	72	81	
10	20	30	40	50	60	70	80	90	100

Use the short multiplication table to find the following products. Then circle the products on the short multiplication table.

$4 \times 4 =$ _____ $4 \times 5 =$ _____

$9 \times 4 =$ _____ $2 \times 5 =$ _____

$8 \times 7 =$ _____ $7 \times 8 =$ _____

$5 \times 7 =$ _____ $3 \times 9 =$ _____

$9 \times 6 =$ _____ $6 \times 9 =$ _____

$7 \times 4 =$ _____ $10 \times 1 =$ _____

Find the following products any way you like.

$\begin{array}{r} 3 \\ \underline{\times\ 4} \end{array}$ $\begin{array}{r} 8 \\ \underline{\times\ 9} \end{array}$ $\begin{array}{r} 6 \\ \underline{\times\ 8} \end{array}$ $\begin{array}{r} 4 \\ \underline{\times\ 6} \end{array}$ $\begin{array}{r} 9 \\ \underline{\times\ 7} \end{array}$ $\begin{array}{r} 2 \\ \underline{\times\ 7} \end{array}$ $\begin{array}{r} 10 \\ \underline{\times\ 4} \end{array}$ $\begin{array}{r} 7 \\ \underline{\times\ 6} \end{array}$ $\begin{array}{r} 5 \\ \underline{\times\ 8} \end{array}$ $\begin{array}{r} 9 \\ \underline{\times\ 9} \end{array}$ $\begin{array}{r} 8 \\ \underline{\times\ 8} \end{array}$

On the short multiplication table, what is special about the last number in each row?

____

How many cells are in row 7? _____ in row 8? _____ in row 5? _____ in row 10? _____

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LESSON 104: AREA OF TANGRAM PIECES

OBJECTIVES:

1. To find the total area by adding the areas of its parts

MATERIALS:

1. Worksheet 84, Area of Tangram Pieces
2. Geared clock
3. A set of tangrams
4. Ruler (for drawing straight lines), optional

ACTIVITIES FOR TEACHING:

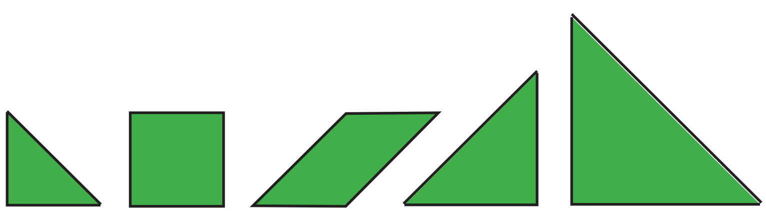
Warm-up. Give the child the worksheet. Tell him to do just the warm-up section. Solutions are:

6834 (**3**)	6834 (**3**)	6834 (**3**)
× 7 (7)	− 4386 (**3**)	+ 4386 (**3**)
28	**2448 (0)**	**11220 (6)**
210		
5600		
42000		
47838 (3)		

Tell the child to say the time set on the geared clock. Include time to the minute, such as 6:03, 2:54, 8:29, and 10:41.

The tangram pieces. Give the child a set of tangrams.

Tell him: Find the smallest triangle. We will call its area 1 unit^2. Ask: What is the area of the other small triangle? [1 unit^2] What is the area of the square? [2 unit^2] How do you know? [Two small triangles fill the square.]



Ask: What is the area of the parallelogram? [2 unit^2] What is the area of the medium triangle? [2 unit^2] What is the area of the large triangle? [4 unit^2] See below.

Ask: What is the total area of all seven pieces? [16 unit^2]

Worksheet 84. Tell him to write the area of the tangram pieces on the worksheet.

Tell him to look at the 10 outlines. Ask: Which ones do you think have the largest area? Tell him: Put a little x near the ones that you think are the largest. You will see how close your guess was when you finish the worksheet.

EXPLANATIONS:

Remember to read "1 unit^2" as "one square unit."

Although area is referred to as "square" units, it is not necessary that it be in the shape of a square. Any two-dimensional shape will work.

Actually, the area of the smallest triangle in the tangram set is very close to 1 in^2 – it is 0.97 in^2.

See page iii, number 15 of "Some General Thoughts on Teaching Mathematics," for additional information.

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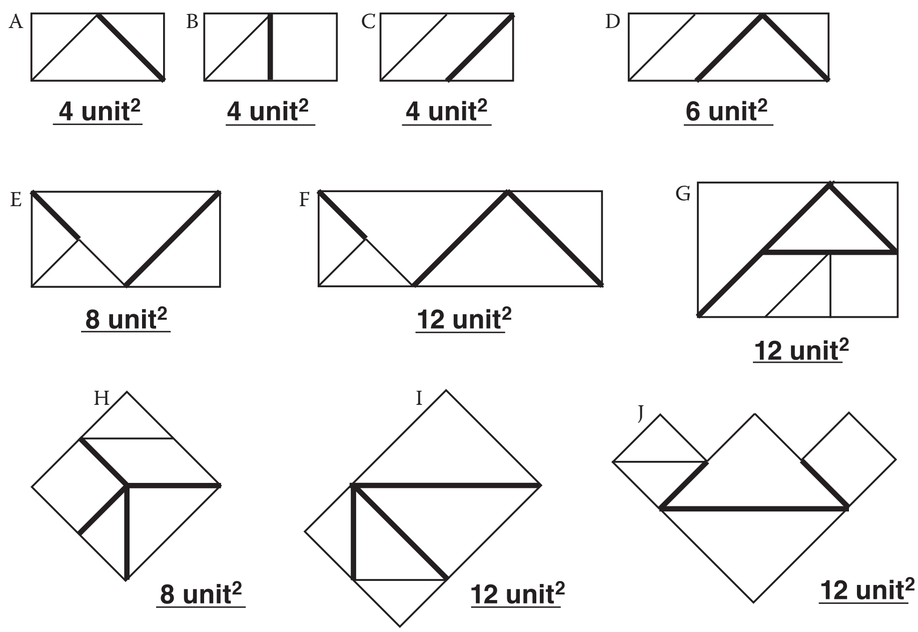
ACTIVITIES FOR TEACHING:	EXPLANATIONS:

Tell him: You are to make the shape shown on the worksheet with the tangram pieces. The first three, A, B, and C, have the same shape, but can be made three different ways. Make the shapes and draw them on your worksheet. Notice the position of the small triangle in each figure. You may use a ruler if you want.

Tell the child to discuss his answers. Ask: What is the area of each figure? [4 unit^2] How did you figure it out? [The two small triangles each has 1 unit^2 and the other piece has 2 unit^2 giving a total of 4 unit^2.]

Tell him to complete the worksheet. Remind him to make each shape with his tangram pieces before copying to the worksheet. Solutions are shown below.



Many of these outlines have more than one solution.

After he has completed the worksheet, ask: Did you guess correctly which shapes had the largest areas?

In conclusion. Ask: Did you notice that the areas of all the figures on the worksheet are even numbers? What would you have to do to make a tangram shape with an odd number of square units? [Use only one of the small triangles.]

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Name: ______________________________

Date: ______________________________

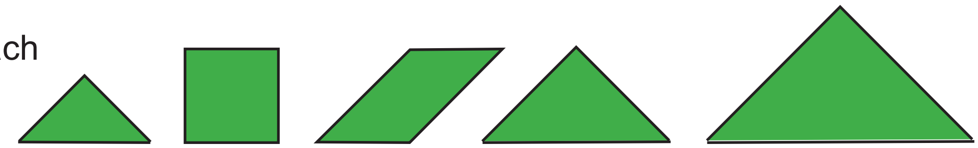
Warm-Up

Multiply 6834 × 7.

Find 6834 – 4386.

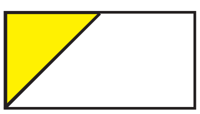
Find 6834 + 4386.

Write the area of each tangram piece.



1 unit^2 ________ ________ ________ ________

Use tangram pieces and draw lines to show the position of the tangram pieces in each figure below. Also give its area in unit^2. Do the first three in different ways.

A

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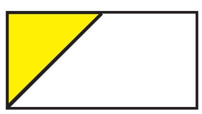
B

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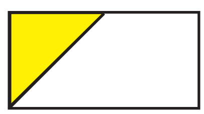
C

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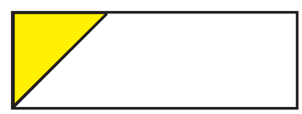
D

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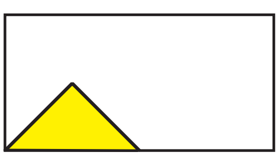
E

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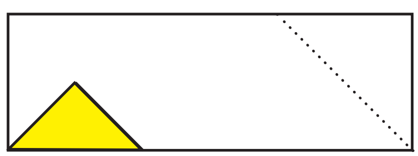
F

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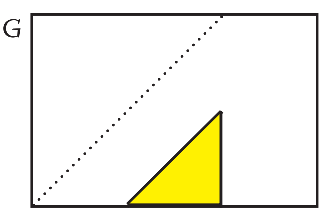
G

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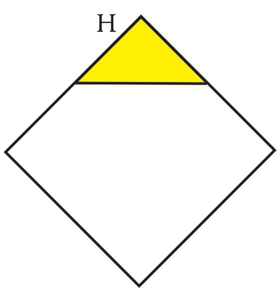
H

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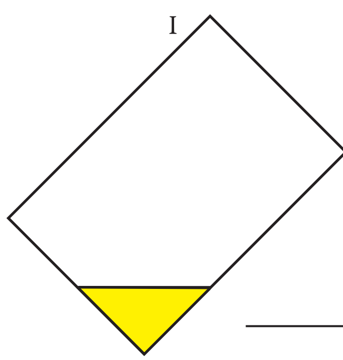
I

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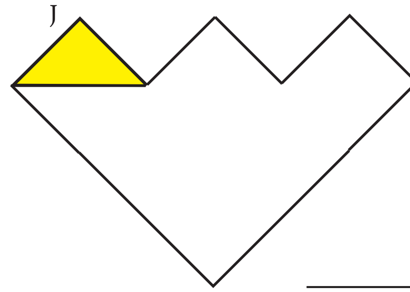
J

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