RightStart™ Mathematics

by Joan A. Cotter, Ph.D.
with Kathleen Cotter Lawler

LEVEL E LESSONS

Second Edition

Activities for Learning, Inc.

A special thank you to Maren Ehley and Rebecca Walsh for their work in the final preparation of this manual.

Note: Levels are used rather than grades. For example, Level A is kindergarten and Level B is first grade and so forth.

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How This Program Was Developed

We have been hearing for years that Japanese students do better than U.S. students in math in Japan. The Asian students are ahead by the middle of first grade. And the gap widens every year thereafter.

Many explanations have been given, including less diversity and a longer school year. Japanese students attend school 240 days a year.

A third explanation given is that the Asian public values and supports education more than we do. A first grade teacher has the same status as a university professor. If a student falls behind, the family, not the school, helps the child or hires a tutor. Students often attend after-school classes.

A fourth explanation involves the philosophy of learning. Asians and Europeans believe anyone can learn mathematics or even play the violin. It is not a matter of talent, but of good teaching and hard work.

Although these explanations are valid, I decided to take a careful look at how mathematics is taught in Japanese first grades. Japan has a national curriculum, so there is little variation among teachers.

I found some important differences. One of these is the way the Asians name their numbers. In English we count ten, eleven, twelve, thirteen, and so on, which doesn't give the child a clue about tens and ones. But in Asian languages, one counts by saying ten-1, ten-2, ten-3 for the teens, and 2-ten 1, 2-ten 2, and 2-ten 3 for the twenties.

Still another difference is their criteria for manipulatives. Americans think the more the better. Asians prefer very few, but insist that they be imaginable, that is, visualizable. That is one reason they do not use colored rods. You can imagine the one and the three, but try imagining a brown eight–the quantity eight, not the color. It cannot be done without grouping.

Another important difference is the emphasis on non-counting strategies for computation. Japanese children are discouraged from counting; rather they are taught to see quantities in groups of fives and tens.

For example, when an American child wants to know 9 + 4, most likely the child will start with 9 and count up 4. In contrast, the Asian child will think that if he takes 1 from the 4 and puts it with the 9, then he will have 10 and 3, or 13. Unfortunately, very few American first-graders at the end of the year even know that 10 + 3 is 13.

I decided to conduct research using some of these ideas in two similar first grade classrooms. The control group studied math in the traditional workbook-based manner. The other class used the lesson plans I developed. The children used that special number naming for three months.

They also used a special abacus I designed, based on fives and tens. I asked 5-year-old Stan how much is 11 + 6. Then I asked him how he knew. He replied, "I have the abacus in my mind."

The children were working with thousands by the sixth week. They figured out how to add 4-digit numbers on paper after learning how on the abacus.

Every child in the experimental class, including those enrolled in special education classes, could add numbers like 9 + 4, by changing it to 10 + 3.

I asked the children to explain what the 6 and 2 mean in the number 26. Ninety-three percent of the children in the experimental group explained it correctly while only 50% of third graders did so in another study.

I gave the children some base ten rods (none of them had seen them before) that looked like ones and tens and asked them to make 48. Then I asked them to subtract 14. The children in the control group counted 14 ones, while the experimental class removed 1 ten and 4 ones. This indicated that they saw 14 as 1 ten and 4 ones and not as 14 ones. This view of numbers is vital to understanding algorithms, or procedures, for doing arithmetic.

I asked the experimental class to mentally add 64 + 20, which only 52% of nine-year-olds on the 1986 National test did correctly; 56% of those in the experimental class could do it.

Since children often confuse columns when taught traditionally, I wrote 2304 + 86 = horizontally and asked them to find the sum any way they liked. Fifty-six percent did so correctly, including one child who did it in his head.

The following year I revised the lesson plans and both first grade classes used these methods. I am delighted to report that on a national standardized test, both classes scored at the 98th percentile.

Joan A. Cotter, Ph.D.

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Some General Thoughts on Teaching Mathematics

1. Only five percent of mathematics should be learned by rote; 95 percent should be understood.
2. Real learning builds on what the child already knows. Rote teaching ignores it.
3. Contrary to the common myth, "young children can think both concretely and abstractly. Development is not a kind of inevitable unfolding in which one simply waits until a child is cognitively 'ready.'" —*Foundations for Success* NMAP
4. What is developmentally appropriate is not a simple function of age or grade, but rather is largely contingent on prior opportunities to learn." —Duschl & others
5. Understanding a new model is easier if you have made one yourself. So, a child needs to construct a graph before attempting to read a ready-made graph.
6. Good manipulatives cause confusion at first. If a new manipulative makes perfect sense at first sight, it is not needed. Trying to understand and relate it to previous knowledge is what leads to greater learning. —Richard Behr & others.
7. According to Arthur Baroody, "Teaching mathematics is essentially a process of translating mathematics into a form children can comprehend, providing experiences that enable children to discover relationships and construct meanings, and creating opportunities to develop and exercise mathematical reasoning."
8. Lauren Resnick says, "Good mathematics learners expect to be able to make sense out of rules they are taught, and they apply some energy and time to the task of making sense. By contrast, those less adept in mathematics try to memorize and apply the rules that are taught, but do not attempt to relate these rules to what they know about mathematics at a more intuitive level."
9. Mindy Holte puts learning the facts in proper perspective when she says, "In our concern about the memorization of math facts or solving problems, we must not forget that the root of mathematical study is the creation of mental pictures in the imagination and manipulating those images and relationships using the power of reason and logic." She also emphasizes the ability to imagine or visualize, an important skill in mathematics and other areas.
10. The only students who like flash cards are those who do not need them.
11. Mathematics is not a solitary pursuit. According to Richard Skemp, solitary math on paper is like reading music, rather than listening to it: "Mathematics, like music, needs to be expressed in physical actions and human interactions before its symbols can evoke the silent patterns of mathematical ideas (like musical notes), simultaneous relationships (like harmonies) and expositions or proofs (like melodies)."
12. "More than most other school subjects, mathematics offers special opportunities for children to learn the power of thought as distinct from the power of authority. This is a very important lesson to learn, an essential step in the emergence of independent thinking." —*Everybody Counts*

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13. The role of the teacher is to encourage thinking by asking questions, not giving answers. Once you give an answer, thinking usually stops.
14. Putting thoughts into words helps the learning process.
15. Help the children realize that it is their responsibility to ask questions when they do not understand. Do not settle for "I don't get it."
16. The difference between a novice and an expert is that an expert catches errors much more quickly. A violinist adjusts pitch so quickly that the audience does not hear it.
17. Europeans and Asians believe learning occurs not because of ability, but primarily because of effort. In the ability model of learning, errors are a sign of failure. In the effort model, errors are natural. In Japanese classrooms, the teachers discuss errors with the whole class.
18. For teaching vocabulary, be sure either the word or the concept is known. For example, if a child is familiar with six-sided figures, we can give him the word, hexagon. Or, if he has heard the word, multiply, we can tell him what it means. It is difficult to learn a new concept and the term simultaneously.
19. Introduce new concepts globally before details. This lets the children know where they are headed.
20. Informal mathematics should precede paper and pencil work. Long before a child learns how to add fractions with unlike denominators, she should be able to add one half and one fourth mentally.
21. Some pairs of concepts are easier to remember if one of them is thought of as dominant. Then the non-dominant concept is simply the other one. For example, if even is dominant over odd; an odd number is one that is not even.
22. Worksheets should also make the child think. Therefore, they should not be a large collection of similar exercises, but should present a variety. In RightStart™ Mathematics, they are designed to be done independently.
23. Keep math time enjoyable. We store our emotional state along with what we have learned. A person who dislikes math will avoid it and a child under stress stops learning. If a lesson is too hard, stop and play a game. Try the lesson again later.
24. In Japan students spend more time on fewer problems. Teachers do not concern themselves with attention spans as is done in the U.S.
25. In Japan the goal of the math lesson is that the student has understood a concept, not necessarily has done something (a worksheet).
26. The calendar must show the entire month, so the children can plan ahead. The days passed can be crossed out or the current day circled.
27. A real mathematical problem is one in which the procedures to find the answer is not obvious. It is like a puzzle, needing trial and error. Emphasize the satisfaction of solving problems and like puzzles, of not giving away the solution to others.

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RightStart™ Mathematics

Ten major characteristics make this research-based program effective:

1. Refers to quantities of up to 5 as a group; discourages counting individually. Uses fingers and tally sticks to show quantities up to 10; teaches quantities 6 to 10 as 5 plus a quantity, for example 6 = 5 + 1.
2. Avoids counting procedures for finding sums and differences. Teaches five- and ten-based strategies for the facts that are both visual and visualizable.
3. Employs games, not flash cards, for practice.
4. Once quantities 1 to 10 are known, proceeds to 10 as a unit. Temporarily uses the "math way" of naming numbers; for example, "1 ten-1" (or "ten-1") for eleven, "1-ten 2" for twelve, "2-ten" for twenty, and "2-ten 5" for twenty-five.
5. Uses expanded notation (overlapping) place-value cards for recording tens and ones; the ones card is placed on the zero of the tens card. Encourages a child to read numbers starting at the left and not backward by starting at the ones.
6. Proceeds rapidly to hundreds and thousands using manipulatives and place-value cards. Provides opportunities for trading between ones and tens, tens and hundreds, and hundreds and thousands with manipulatives.
7. Teaches mental computation. Investigates informal solutions, often through story problems, before learning procedures.
8. Teaches four-digit addition on the abacus, letting the child discover the paper and pencil algorithm.
9. Introduces fractions with a linear visual model, including all fractions from 1/2 to 1/10. "Pies" are not used initially because they cannot show fractions greater than 1. Later, the tenths will become the basis for decimals.
10. Teaches short division (where only the answer is written down) for single-digit divisors, before long division.

Second Edition

Many changes have occurred since the first RightStart™ lessons were begun in 1994. First, mathematics is used more widely in many fields, for example, architecture, science, technology, and medicine. Today, many careers require math beyond basic arithmetic. Second, research has given us new insights into how children learn mathematics. Third, kindergarten has become much more academic, and fourth, most children are tested to ensure their preparedness for the next step.

This second edition is updated to reflect new research and applications. Topics within a grade level are always taught with the most appropriate method using the best approach with the child and teacher in mind.

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Daily Lessons

Objectives. The objectives outline the purpose and goal of the lesson. Some possibilities are to introduce, to build, to learn a term, to practice, or to review.

Materials. The Math Set of manipulatives includes the specially crafted items needed to teach RightStart™ Mathematics. Occasionally, common objects such as scissors will be needed. These items are indicated by boldface type.

Warm-up. The warm-up time is the time for quick review, memory work, and sometimes an introduction to the day's topics. The dry erase board makes an ideal slate for quick responses.

Activities. The Activities for Teaching section is the heart of the lesson; it starts on the left page and continues to the right page. These are the instructions for teaching the lesson. The expected answers from the child are given in square brackets.

Establish with the children some indication when you want a quick response and when you want a more thoughtful response. Research shows that the quiet time for thoughtful response should be about three seconds. Avoid talking during this quiet time; resist the temptation to rephrase the question. This quiet time gives the slower child time to think and the quicker child time to think more deeply.

Encourage the child to develop persistence and perseverance. Avoid giving hints or explanations too quickly. Children tend to stop thinking once they hear the answer.

Explanations. Special background notes for the teacher are given in Explanations.

Worksheets. The worksheets are designed to give the children a chance to think about and to practice the day's lesson. The children are to do them independently. Some lessons, especially in the early levels, have no worksheet.

Games. Games, not worksheets or flash cards, provide practice. The games, found in the *Math Card Games* book, can be played as many times as necessary until proficiency or memorization takes place. They are as important to learning math as books are to reading. The *Math Card Games* book also includes extra games for the child needing more help, and some more challenging games for the advanced child.

In conclusion. Each lesson ends with a short summary called, "In conclusion," where the child answers a few short questions based on the day's learning.

Number of lessons. Generally, each lesson is be done in one day and each manual, in one school year. Complete each manual before going on to the next level. Other than Level A, the first lesson in each level is an introductory test with references to review lessons if needed.

Comments. We really want to hear how this program is working. Please let us know any improvements and suggestions that you may have.

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Lesson 56: Adding Mixed Fractions with Eighths

OBJECTIVES:

1. To learn the terms *proper fraction* and *improper fraction*
2. To practice adding fractions with eighths
3. To convert improper eighths to proper eighths

MATERIALS:

1. Fraction chart and fraction pieces
2. *Math Card Games* book, F22.1
3. Math journal

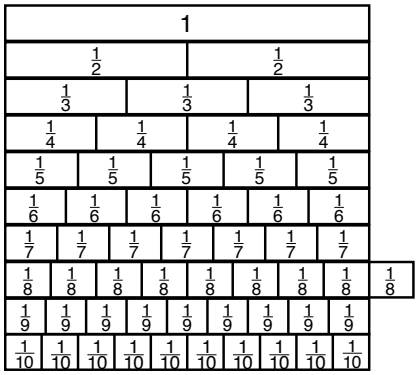
ACTIVITIES FOR TEACHING:

Warm-up. Ask: In the fraction one fifth, what is the denominator? [5] In the fraction one fifth, what number is the numerator? [1] If the denominator and numerator are the same, what does the fraction equal? [1]

Improper fractions. Give the child the fraction chart and fraction pieces.

Write: $\frac{9}{8}$

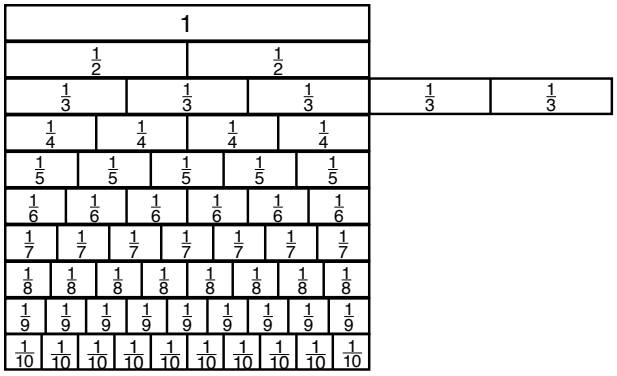
and ask the child to show it with her fraction chart and fraction pieces. [8 eighths plus 1 more eighth] See figure below.



Showing $\frac{9}{8}$ with the fraction chart and pieces.

Write: $\frac{5}{3}$

and tell her to show it with the fraction materials. See figure below.



Showing $\frac{5}{3}$ with the fraction chart and pieces.

EXPLANATIONS:

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ACTIVITIES FOR TEACHING CONTINUED:

Write: $\frac{9}{8}$ $\frac{5}{3}$ $\frac{3}{4}$

Ask: Which of these three fractions is less than one? [$\frac{3}{4}$] How can you tell by looking only at the numerators and denominators? [The numerator is less than the denominator.]

Say: When the numerator is less than the denominator, the fraction is called a *proper fraction*. This name results from hundreds of years ago when people thought a "real" fraction had to be less than one. The word "fraction" comes from the Latin word "frangere" meaning "to break." Two other words from this root word are fracture and fragment. Mathematicians realized fractions were division and often were not less than one. They called fractions equal to or greater than one *improper fractions*.

Write: $\frac{4}{8}$ $\frac{7}{4}$ $\frac{4}{3}$ $\frac{8}{8}$ $\frac{12}{8}$ $\frac{1}{6}$

Ask: Which of these are proper fractions? [only the first and last fractions, $\frac{4}{8}$ and $\frac{1}{6}$]

Ask: How can we rewrite the improper fractions using a whole number plus a fraction? [$1\frac{3}{4}$, $1\frac{1}{3}$, 1, $1\frac{4}{8}$]

Preparation for Corners™ with Eighths game. Explain that the game for the day will be a Corners™ Three game variation. Now each number on the cards will be *eighths*. For example, 3 is $\frac{3}{8}$ and 9 is $\frac{9}{8}$.

Write: $1\frac{3}{8} + \frac{9}{8} =$

and ask the child to add it. Ask the child to explain her work.

One way is: $1\frac{3}{8} + \frac{9}{8} = 1\frac{12}{8} = 2\frac{4}{8}$

Another way is: $1\frac{3}{8} + \frac{\cancel{9}^{\,1}}{\cancel{8}} = 2\frac{4}{8}$

Give her another example: $2\frac{\cancel{2}^{\,1}}{\cancel{8}}$

$2\frac{5}{8} + \frac{18}{8} = [2\frac{23}{8} = 4\frac{7}{8}$ or $2\frac{5}{8} + \frac{\cancel{18}^{\,10}}{\cancel{8}} = 4\frac{7}{8}]$

Corners™ with Eighths game. Play Corners™ with Eighths game, found in *Math Card Games* book, F22.1. Stress that the fractions in the scoring sums must be proper fractions. Tell her to write the scoring in her math journal.

In conclusion. Ask: What do we call a fraction when the numerator is greater than the denominator? [improper] What is a fraction called when the denominator is greater than the numerator? [proper]

EXPLANATIONS CONTINUED:

This can be done by referring to the fraction chart. No algorithm is necessary.

The answers need not be in lowest terms.

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Lesson 59: Multiplying by Two Digits

OBJECTIVES:

1. To develop a procedure for multiplying by two digits

MATERIALS:

1. Worksheet 37, Multiplying by Two Digits

ACTIVITIES FOR TEACHING:

Warm-up. Ask: What is 31 × 2? [62] What is 31 × 20? [620] What is 31 × 200? [6200]

Ask: What is 23 × 3? [69] What is 23 × 30? [690] What is 23 × 300? [6900]

Multiplying by two digits. Write these three problems:

312	312	312
× 2	× 30	× 32
624	9360	

Say: You have been multiplying problems like the first one for several months now. In yesterday's lesson you multiplied numbers with tens like the second problem. Today you will multiply numbers with two digits like the third problem.

Ask: How do you think you could do it? Tell the child to share her thoughts. Two solutions are below.

312	312
× 32	× 32
624	9360
9360	624
9984	9984

EXPLANATIONS:

It is acceptable to multiply the leftmost digit first.

Worksheet 37. Give the child the worksheet and tell her to do the first two rows in the left box. The solutions are below.

Then tell her to discuss her answer.

63	63	63	825	825	825
× 5	× 30	× 35	× 6	× 50	× 56
315	**1890**	**315**	**4950**	**41,250**	**4950**
		1890			**41250**
		2205			**46,200**

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ACTIVITIES FOR TEACHING CONTINUED:

Repeat for the last two rows in the left box. The solutions are below.

3674	3674	3674	9062	9062	9062
× 1	× 80	× 81	× 7	× 20	× 27
3674	**293,920**	**3674**	**63,434**	**181,240**	**63434**
		293920			**181240**
		297,594			**244,674**

Writing the 'carries.' Write:

$$\begin{array}{r} 28 \\ \times\ 43 \\ \hline \end{array}$$

and multiply the 28 × 3 part. See below.

$$\begin{array}{r} {\scriptstyle 2} \\ 28 \\ \times\ 43 \\ \hline 84 \end{array}$$

Continue with multiplying the 28 × 40.

$$\begin{array}{r} {\scriptstyle 3} \\ {\scriptstyle 2} \\ 28 \\ \times\ 43 \\ \hline 84 \\ 1120 \\ \hline 1204 \end{array}$$

Explain that the carries, the little numbers, can be written in rows above the problem, but many people do not write them at all; they do it mentally.

Worksheet 37. Tell the child to complete the worksheet. The solutions are below.

81	143	572	2927	
× 52	× 33	× 64	× 81	
162	**429**	**2288**	**2927**	
4050	**4290**	**34320**	**234160**	
4212	**4719**	**36,608**	**237,087**	
365	**365**	**365**	**365**	**365**
× 2	**× 9**	**× 10**	**× 55**	**× 26**
730	**3285**	**3650**	**1825**	**2190**
			18250	**7300**
			20,075	**9490**

In conclusion. Ask: If you multiply 2 by 50 and then 2 by 3 and add them together, what is the answer? [106, 2 × 53] If you multiply any number by 50 and then by 3 and add them together, what is the answer? [number × 53]

EXPLANATIONS CONTINUED:

Do not insist that the child write the little ones. Some can do it mentally.

Technically, it is not necessary to write the 0 in the right column of the second line. However, it helps children in their understanding that they are multiplying by 3 tens and not by 3 ones.

Unfortunately, some children have been taught to write an "x" as the placeholder. This nonstandard use of x has caused those children considerable confusion when they studied algebra.

If there is additional time following this lesson, play the Multiples Solitaire game, found in *Math Card Games* book, P19.

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Name: _____

Date: _____

1. Find the products by using your previous answers wherever possible.

63 × 5	63 × 30	63 × 35
825 × 6	825 × 50	825 × 56
3674 × 1	3674 × 80	3674 × 81
9062 × 7	9062 × 20	9062 × 27

2. Multiply.

81 × 52	143 × 33	572 × 64
		2927 × 81

3. How many days are in the following number of years? Ignore leap years.

2 yr	9 yr	10 yr	55 yr	26 yr
365				

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Lesson 78: Using Decimal Points for Hundredths

OBJECTIVES:

1. To understand decimals as an alternate way of writing tenths and hundredths
2. To subtract tenths and hundredths in decimal format

MATERIALS:

1. Warm-up Practice 3
2. AL Abacus and about 10 centimeter cubes
3. Place-value cards
4. *Math Card Games* book, N43 and F22.2, and Math journal
5. Worksheet 51, Using Decimal Points for Hundredths

ACTIVITIES FOR TEACHING:

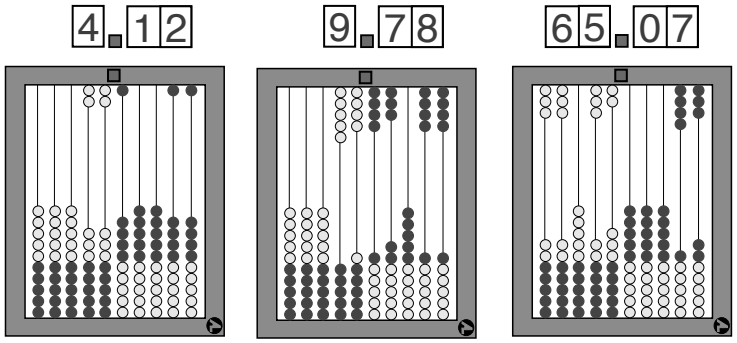
Warm-up. Give the child the warm-up practice sheet. Tell him to do the second multivide on the page. Solutions are on the right.

Writing hundredths as decimals. Give the child the abacus, centimeter cubes, and place-value cards.

Write: $4\frac{12}{100}$

and ask: How do you think you could write this using a decimal point? Write it: 4.12

Say: We read it as 4 and 12 hundredths. Compose the number with your place-value cards and enter it on your abacus. See the left figure below.



Entering 4.12. **Entering 9.78.** **Entering 65.07.**

Repeat for nine and 78 hundredths. See the middle figure above.

Repeat for 65 and 7 hundredths. (To get the zero, turn a tens card for example, the 30-card, upside down and cover the 3 with the 7.) See the right figure above. Ask: Why did you need a zero before the 7? [Without it, it would be 65 and 7 tenths.]

Practice. Write and ask him to read the following:

30.72 [30 and 72 hundredths]

72.8 [72 and 8 tenths]

72.08 [72 and 8 hundredths]

9.40 [9 and 40 hundredths]

EXPLANATIONS:

5678 (**8**)	5678 (**8**)	5678 (**8**)
× 2 (**2**)	× 70 (**7**)	× 72 (**0**)
11 356 (**7**)	**397 460** (**2**)	**11 356**
		397 460
		408 816 (**0**)
	213 459 (**6**)	
	× 35 (**8**)	
	1 **067 295**	
	6 403 770	
	7 471 065 (**3**)	

This question encourages the child to think of the big picture and to continue to think intuitively about math.

Do not at this point read 4.12 as "four point one two." This lesson is to help the child understand the relationship between fractions and decimals.

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ACTIVITIES FOR TEACHING CONTINUED:

Can You Find game. Play this variation of the Can You Find game, found in *Math Card Games* book, N43. Use place-value cards with ones and tens and seven centimeter cubes. Below are the numbers to say. Tell the child to compose the number and set it aside. All the cards will be collected at the end of the game.

1. Can you find 90 and 5 tenths?
2. Can you find 8 tenths?
3. Can you find 60 and 87 hundredths?
4. Can you find 50 and 12 hundredths?
5. Can you find 24 and 3 tenths?
6. Can you find 71 and 36 hundredths?
7. Can you find 9 hundredths? (Hint: Turn the 40-card upside down to get a zero.)

Subtracting tenths and hundredths. Write:

4.1	2.37	3.26
– .3	– 1.31	– 1.48

and ask the child to find the differences any way he can. [3.8, 1.06, 1.78] He could do it with the abacus or by thinking in terms in tenths and hundredths as fractions.

Worksheet 51. Give the child the worksheet and tell him to do the problems. The solutions are below.

$1\frac{29}{100}$ 1.29 **$52\frac{52}{100}$ 52.52** **$63\frac{49}{100}$ 63.49**

$\frac{83}{100}$ .83 **$8\frac{7}{100}$ 8.07** **$8\frac{9}{10}$ 8.9**

21.6	9.3	10.0	9.1
– 3.5	– 5.6	– 8.5	– 8.3
18.1	**3.7**	**1.5**	**.8**
11.63	9.47	9.53	7.41
– 2.31	– 2.87	– 5.28	– 5.53
9.32	**6.60**	**4.25**	**1.88**
5.68	5.15	8.00	3.40
– 2.08	– 2.90	– 1.25	– 1.25
3.60	**2.25**	**6.75**	**2.15**

Corners™ with Tenths game. Play this variation of Corners™ with Tenths game found in Math Card Games book, F22.2. Say: Use your math journal to write the scores using decimal points. In this game, all the numbers are considered to be hundredths. A score of 12 is now 12 hundredths, written with a decimal point (.12). Tell him to use his math journal for scoring.

In conclusion. Ask: Which is more, 7 tenths or 7 hundredths? [7 tenths] Which is more, 7 tenths or 70 hundredths? [the same] Which is more, 7 or 7 tenths? [7]

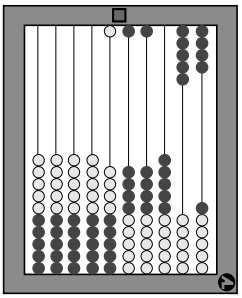
EXPLANATIONS CONTINUED:

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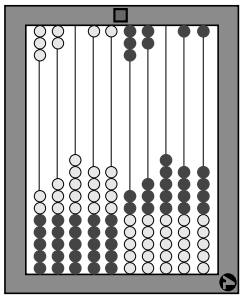
Name: _____

Date: _____

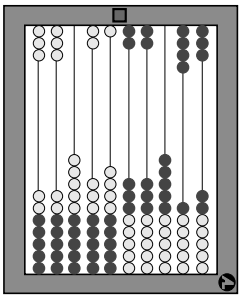
Write the quantities shown using fractions and decimal points.



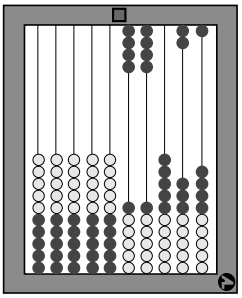
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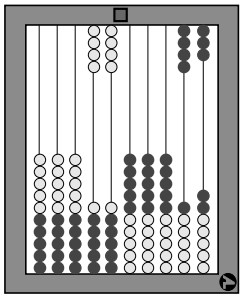
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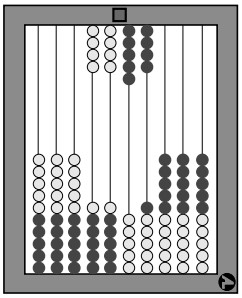
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_____ _____



_____ _____

Subtract the following.

21.6 − 3.5	9.3 − 5.6	10.0 − 8.5	9.1 − 8.3
11.63 − 2.31	9.47 − 2.87	9.53 − 5.28	7.41 − 5.53
5.68 − 2.08	5.15 − 2.90	8.00 − 1.25	3.40 − 1.25

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Lesson 95: More Percentage Problems

OBJECTIVES:

1. To solve more common problems involving percentages
2. To learn about tipping and sales tax

MATERIALS:

1. Warm-up Practice 9
2. Worksheet 67, More Percentage Problems
3. *Math Card Games* book, F48

ACTIVITIES FOR TEACHING:

Warm-up. Give the child the warm-up practice sheet. Tell him to do the second multivide on the page. Solutions are on the right.

Worksheet 70. Give the child the worksheet and ask him to read and solve the first problem. Then tell him to explain it.

1. In a certain class 50% of the children are girls. There are 12 girls. How many children are in the class? [24 children]

If 50% are girls, then 50% must be boys. The total number will be 12 × 2 = 24 children.

Repeat for the remaining problems.

2. The usual tip at a restaurant is 15% of the cost of the food. Many people figure it out by first finding 10%, then finding 5%, which is half of 10%, and adding them together. What is the tip if the food costs $8.00? [$1.20]

Ten percent of $8 is $0.80. Half of that is $0.40. Adding $0.80 and $0.40 is $1.20.

3. What is the 15% tip if the food bill is $12.00? What is the total cost? [$13.80]

Tip is $1.20 + $0.60 = $1.80. Total is $12 + $1.80 = $13.80.

4. In some places people pay sales tax on certain things they buy. If the sales tax is 5%, what is the total bill for a car that cost $4000? [$4200]

Ten percent of $4000 is $400. Half of that is $200. Total is $4000 + $200 = $4200 total cost.

EXPLANATIONS:

```
          24 (6)
      × 18 (0)
         192
         240
         432 (0)
       × 6 (6)
       2 592 (0)
      × 35 (8)
      12 960
      77 760
      90 720 (0)
      × 96 (6)
      444 320
    8 164 800
9) 8 709 120 (0)
8) 967 680 (0)
7) 120 960 (0)
6) 17 280 (0)
5) 2 880 (0)
4) 576 (0)
3) 144 (0)
2) 48 (3)
      24
```


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ACTIVITIES FOR TEACHING CONTINUED:

5. The original price for a game is $10.00. In Store A it went on sale at 10% off and then it went on sale again with 50% off of the sale price. In Store B it went on sale at 50% off and then it went on sale again with 10% off of the sale price. Which store has the better price? [the same, $4.50]

At Store A, the price after the first reduction is $10 × 90% = $9. After the second price reduction, it is $9 × 50% = $4.50.

At Store B, the price after the first reduction is $10 × 50% = $5. After the second price reduction, it is $5 × 90% = $4.50.

Percentage War game. Have him play the Percentage War game, found in *Math Card Games* book, F48.

In conclusion. Ask: Which is more, one half or 60%? [60%] Which is more, three eighth or 20%? [three eighths] Which is more, two thirds or four fifths? [four fifths]

EXPLANATIONS CONTINUED:

Note that the final price for Store A is $10 × 50% × 90% and for Store B it is $10 × 90% × 50%, which gives the same result.

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Lesson 100: Measuring Angles

OBJECTIVES:

1. To introduce the term *angle*
2. To measure angles with the goniometer
3. To measure and add the angles in a triangle

MATERIALS:

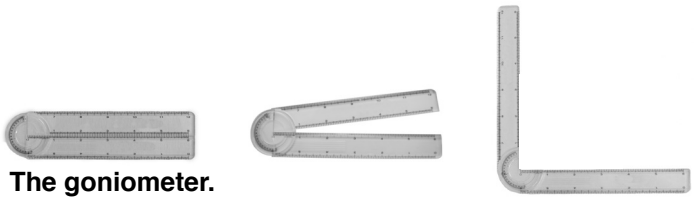
1. Warm-up Practice 10
2. Goniometer
3. 45 triangle and 30-60 triangle
4. Worksheet 72, Measuring Angles

ACTIVITIES FOR TEACHING:

Warm-up. Give the child the warm-up practice sheet. Tell him to do only the first multivide. Solutions are on the right.

The goniometer. Give the child the goniometer and triangles. Tell him that a goniometer (GON-ee-OM-i-ter) measures the *angles*. An angle is the space between two lines at their vertex, or intersecting point.

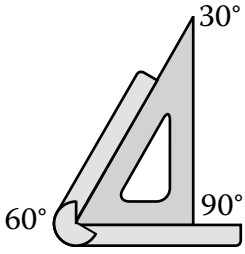
Lay the goniometer flat on a surface and demonstrate how to open it by holding the bottom part with your right hand and gently opening the top part with your left hand. See the left figure below.



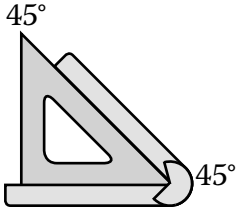
The goniometer.

Tell him to open his goniometer so the inside edges are perpendicular to make a right angle. See the right figure above. Tell him to look at the number inside the little magnifying bubble and ask: What number do you see? [90] Tell him to read it as 90 degrees. Tell him to continue to open the goniometer to twice 90°. Ask: What is the angle? [180°]

Measuring angles. Tell him to measure the angles in the triangles with his goniometer. See the figures below.



Measuring 60°.



Measuring 45°.

Ask: Which triangle has two angles that are congruent, or the same? [45 triangle]

EXPLANATIONS:

Goniometers were briefly introduced in Level D, Lesson 116.

If the two parts of the goniometer come apart, they can be snapped back together. Align the part with the bump on top of the other part and press down.

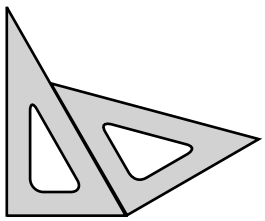
```
                22 (4)
             × 27 (0)
               154
               440
               594 (0)
             × 35 (8)
             2 970
            17 820
            20 790 (0)
             × 24 (6)
            83 160
           415 800
           498 960 (0)
             × 16 (7)
         2 993 760
         4 989 600
9)7 983 360 (0)
8)887 040 (0)
7)110 880 (0)
6)15 840 (0)
5)2 640 (3)
4)528 (6)
3)132 (6)
2)44 (8)
          22
```

Congruent is defined as fitting exactly on top.

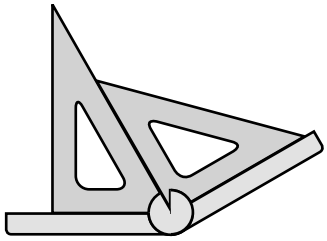
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ACTIVITIES FOR TEACHING CONTINUED:

Combining angles. Tell him to place the 90° angle of the 45 triangle next to the 60° angle of the 30-60 triangle. See the left figure shown below. Ask: What do you think the angle is now? [60 + 90 = 150°] Tell him to use his goniometer to check. [150°] See the right figure below.

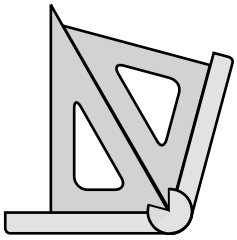


60° + 90°

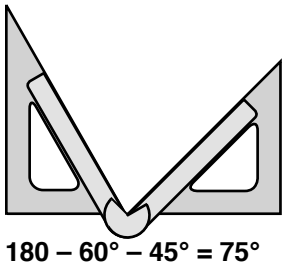


Measuring 150°.

Tell him to place the 45° angle next to the 60° angle. Ask: What is the combined angle? [105°] Tell him to use his goniometer to check. [105°] See the left figure below.



60° + 45° = 105°.

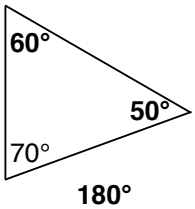


180 – 60° – 45° = 75°

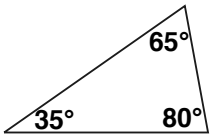
Tell him to place the two triangles on a straight line with the right angles on the outside as in the right figure above. Ask: How could you find the angle between them? [180 – 60° – 45° = 75°] Tell him to check with his goniometer. [75°]

Worksheet 76. Give the child the worksheet and tell him to complete it. He will need a goniometer. The solutions are below.

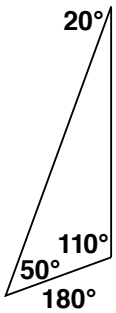
45 + 60 = 105°	**45 + 30 = 75°**	**90 + 90 = 180°**
45 + 90 = 135°	**30 + 90 = 120°**	**90 + 60 = 150°**
180 – 45 – 60 = 75°	**45 – 30 = 15°**	**60 – 45 = 15°**



180°



180°



180°

In conclusion. Ask: What are the angles in the 45 triangle? [45°, 45°, 90°] What are the angles in the 30-60 triangle? [30°, 60°, 90°] How many degrees are in a right angle? [90°]

EXPLANATIONS CONTINUED:

If the worksheets are coil bound, it may be easier to remove the page so that the goniometer lays flat on the page.

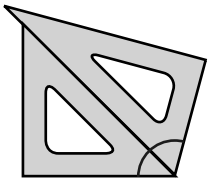
If there is additional time following this lesson, play the Subtracting from One Hundred game, found in *Math Card Games* book, S33.

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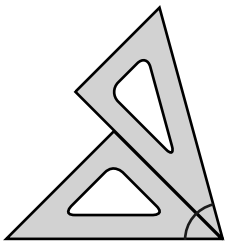
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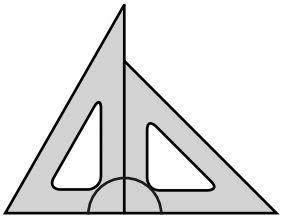
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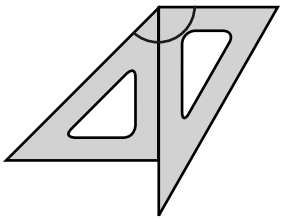
For each figure below, calculate the angle identified by the arc. Then check it with a goniometer.

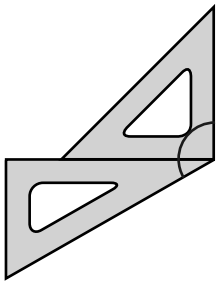


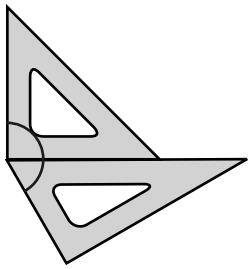
45 + 60 = 105° ✓

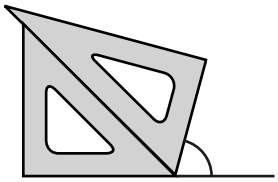


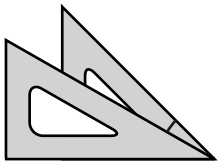


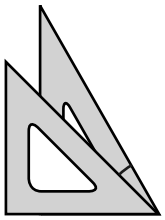




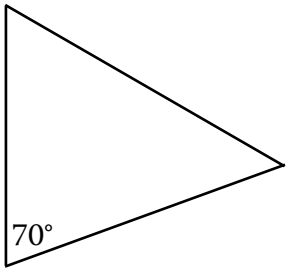


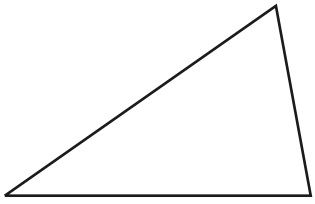


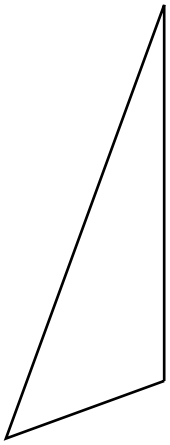




For each triangle, measure the angles and add them up.







Lesson 122: Isometric Drawings

OBJECTIVES:

1. To introduce isometric drawing
2. To practice visualizing objects
3. To make some simple isometric drawings

MATERIALS:

1. Warm-up Practice 12
2. Worksheet 94, Isometric Drawings
3. 35 centimeter cubes
4. Drawing board
5. T-square and 30-60 triangle
6. 10 tiles

ACTIVITIES FOR TEACHING:

Warm-up. Give the child the warm-up practice sheet. Tell him to do the second multivide on the page. Solutions are on the right.

Worksheet 94. Give the child the worksheet, centimeter cubes, drawing board, T-square, triangle, and tiles. Tell him to tape the worksheet to his drawing board.

Problem 1. Tell the child to read the instructions on the worksheet for Problem 1 then to use his triangle to find the angles of the lines. [90° and 30°]

Explain that the word "isometric" (i-so-MET-ric) comes from two Greek words, "iso" meaning "equal" and "metric" meaning "measure." Ask: What other mathematical word starts with "iso"? [isosceles] What does isosceles mean? [equal legs]

Ask: What small figures makes up the background for the isometric drawings? [equilateral triangles] What is special about them? [All three sides are equal.] Say: This means that the units are the same in each direction. Isometric drawings are a way to show three dimensions on a flat surface.

Tell the child that the terms *width*, *length*, and *height* do not have exact definitions. Sometimes *breadth* and *depth* are also used. Because of possible confusion, companies that sell boxes do not use these words to describe the dimensions of their boxes, but use drawings or just the measurements instead.

Tell him to make a cube with his centimeter cubes that measures 2 cm on a side. See the left figure on the next page. Then tell him to make another cube that measures 3 cm on a side. See the right figure. Ask: How does the length, width, and height change? [increases by 1 cm]

EXPLANATIONS:

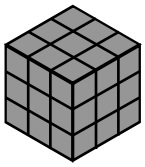
```
          16 (7)
      × 90 (0)
    1 440 (0)
       × 56 (2)
     8 640
    72 000
    80 640 (0)
       × 72 (0)
   161 280
 5 644 800
9)5 806 080 (0)
8)645 120 (0)
7)80 640 (0)
6)11 520 (0)
5)1 920 (3)
4)384 (6)
3)96 (6)
2)32 (5)
  16
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ACTIVITIES FOR TEACHING CONTINUED:

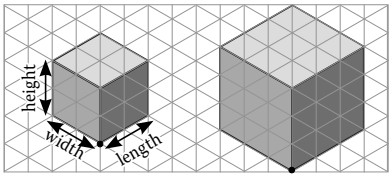


Cube with 2 cm side.

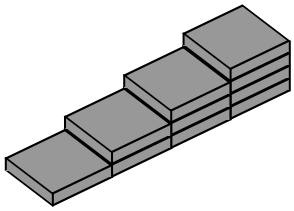


Cube with 3 cm side.

Tell him to draw the 3 cm cube for Problem 1. The solution is shown below.

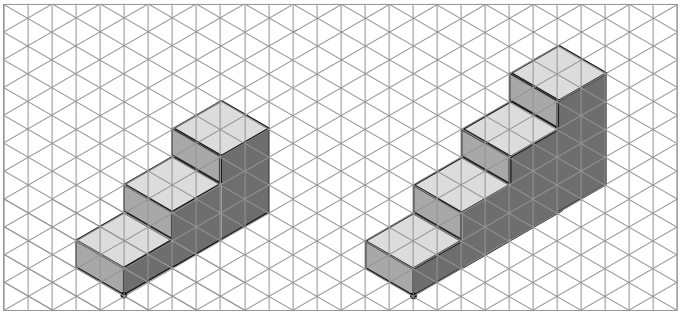


Problem 2. Tell the child to read the instructions for the second problem. Tell him to make the stairs he needs with tiles first. See the figure below.

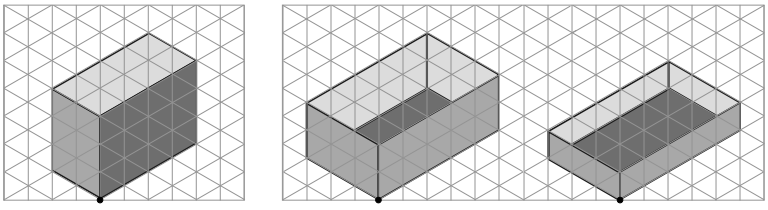


The stairs built with tiles.

Then tell him to draw the stairs. The solution is shown below. Tell him to explain his solution.



Problems 3 and 4. Tell him to complete the worksheet. The solutions are below.



In conclusion. Ask: Do you see any rectangular prisms in the room? [possibly a brick, book, picture frame, box, table top, and window glass.]

EXPLANATIONS CONTINUED:

Shading isn't strictly necessary, but it makes the figure more realistic.

The child will need this worksheet for the next lesson.

If there is additional time following this lesson, play the Card Exchange game, found in *Math Card Games* book, P27.

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