RightStart™ Mathematics

by Joan A. Cotter, Ph.D.
with Kathleen Cotter Lawler

LEVEL F LESSONS

Second Edition

Activities for Learning, Inc.

A special thank you to Maren Ehley, Rebecca Walsh, and Kelsie Burza for their work in the final preparation of this manual.

Note: Levels are used rather than grades. For example, Level A is kindergarten and Level B is first grade and so forth.

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RightStart™ Mathematics

Ten major characteristics make this research-based program effective:

1. Refers to quantities of up to 5 as a group; discourages counting individually. Uses fingers and tally sticks to show quantities up to 10; teaches quantities 6 to 10 as 5 plus a quantity, for example 6 = 5 + 1.
2. Avoids counting procedures for finding sums and differences. Teaches five- and ten-based strategies for the facts that are both visual and visualizable.
3. Employs games, not flash cards, for practice.
4. Once quantities 1 to 10 are known, proceeds to 10 as a unit. Temporarily uses the "math way" of naming numbers; for example, "1 ten-1" (or "ten-1") for eleven, "1-ten 2" for twelve, "2-ten" for twenty, and "2-ten 5" for twenty-five.
5. Uses expanded notation (overlapping) place-value cards for recording tens and ones; the ones card is placed on the zero of the tens card. Encourages a child to read numbers starting at the left and not backward by starting at the ones.
6. Proceeds rapidly to hundreds and thousands using manipulatives and place-value cards. Provides opportunities for trading between ones and tens, tens and hundreds, and hundreds and thousands with manipulatives.
7. Teaches mental computation. Investigates informal solutions, often through story problems, before learning procedures.
8. Teaches four-digit addition on the abacus, letting the child discover the paper and pencil algorithm.
9. Introduces fractions with a linear visual model, including all fractions from 1/2 to 1/10. "Pies" are not used initially because they cannot show fractions greater than 1. Later, the tenths will become the basis for decimals.
10. Teaches short division (where only the answer is written down) for single-digit divisors, before long division.

Second Edition

Many changes have occurred since the first RightStart™ lessons were begun in 1994. First, mathematics is used more widely in many fields, for example, architecture, science, technology, and medicine. Today, many careers require math beyond basic arithmetic. Second, research has given us new insights into how children learn mathematics. Third, kindergarten has become much more academic, and fourth, most children are tested to ensure their preparedness for the next step.

This second edition is updated to reflect new research and applications. Topics within each level are always taught with the most appropriate method using the best approach with the child and teacher in mind.

Daily Lessons

Objectives. The objectives outline the purpose and goal of the lesson. Some possibilities are to introduce, to build, to learn a term, to practice, or to review.

Materials. The Math Set of manipulatives includes the specially crafted items needed to teach RightStart™ Mathematics. Occasionally, common objects such as scissors will be needed. These items are indicated by boldface type.

Warm-up. The warm-up time is the time for quick review, memory work, and sometimes an introduction to the day's topics. The dry erase board makes an ideal slate for quick responses.

Activities. The Activities for Teaching section is the heart of the lesson; it starts on the left page and continues to the right page. These are the instructions for teaching the lesson. The expected answers from the child are given in square brackets.

Establish with the children some indication when you want a quick response and when you want a more thoughtful response. Research shows that the quiet time for thoughtful response should be about three seconds. Avoid talking during this quiet time; resist the temptation to rephrase the question. This quiet time gives the slower child time to think and the quicker child time to think more deeply.

Encourage the child to develop persistence and perseverance. Avoid giving hints or explanations too quickly. Children tend to stop thinking once they hear the answer.

Explanations. Special background notes for the teacher are given in Explanations.

Worksheets. The worksheets are designed to give the children a chance to think about and to practice the day's lesson. The children are to do them independently. Some lessons, especially in the early levels, have no worksheet.

Games. Games, not worksheets or flash cards, provide practice. The games, found in the *Math Card Games* book, can be played as many times as necessary until proficiency or memorization takes place. They are as important to learning math as books are to reading. The *Math Card Games* book also includes extra games for the child needing more help, and some more challenging games for the advanced child.

In conclusion. Each lesson ends with a short summary called, "In conclusion," where the child answers a few short questions based on the day's learning.

Number of lessons. Generally, each lesson is to be done in one day and each manual, in one school year. Complete each manual before going on to the next level.

Comments. We really want to hear how this program is working. Please let us know any improvements and suggestions that you may have.

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Lesson 24: Tenths and Hundredths

OBJECTIVES:

1. To review tenths and hundredths
2. To add and subtract tenths and hundredths

MATERIALS:

1. Worksheet 13, Tenths and Hundredths
2. AL Abacus
3. *Math Card Games* book, S11
4. Math journal

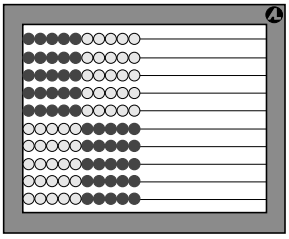
ACTIVITIES FOR TEACHING:

Warm-up. Give the child the worksheet. Tell her to complete just the warm-up problems. Solutions are below.

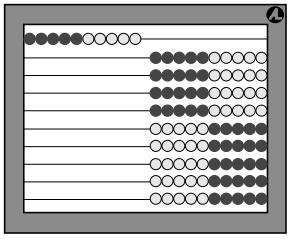
$1^5 + 2^4 + 3^3 =$ **44** $\quad 3^3 - 2^4 - 1^5 =$ **10**

$\frac{3^3}{1^5} - 2^4 =$ **11** $\quad \frac{1^5}{3^3} + 2^4 =$ **16$\frac{1}{27}$**

Reviewing tenths. Give the child the abacus. Tell her that now all one hundred beads on the abacus will be considered to be one. Tell her to enter one. See the left figure below.



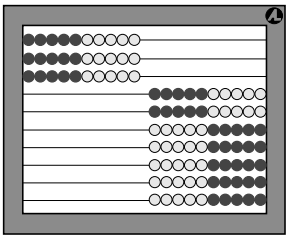
Representing 1.



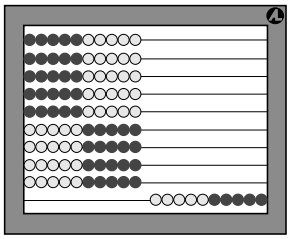
Representing 0.1.

Now tell her to enter one tenth. See the right figure above. Ask: What are two ways to write one tenth? [$\frac{1}{10}$ and 0.1]

Tell her to enter three tenths. See the left figure below. Ask: What are two ways to write it? [$\frac{3}{10}$ and 0.3] Repeat for nine tenths. [$\frac{9}{10}$ and 0.9] See the right figure below.



Representing 0.3.



Representing 0.9.

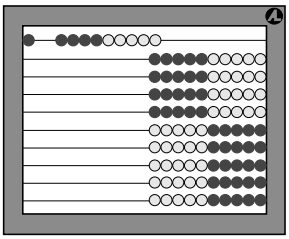
Reviewing Hundredths. Ask: What is one tenth of one tenth? [one hundredth] Tell her to show one hundredth on her abacus. See the left figure on the next page.

EXPLANATIONS:

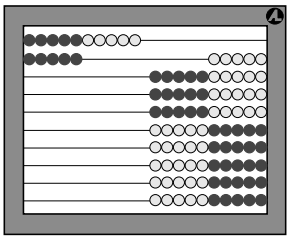
This lesson is similar to Lesson 74 in RightStart™ Mathematics Level E Second Edition, however it provides a foundation for the upcoming lessons.

ACTIVITIES FOR TEACHING CONTINUED:

Tell her to enter one tenth and five hundredths. See the right figure below. Ask: How many hundredths is this? [15 hundredths]

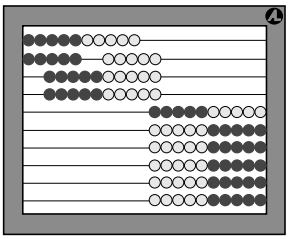


One tenth of one tenth is one hundredth, 0.01.

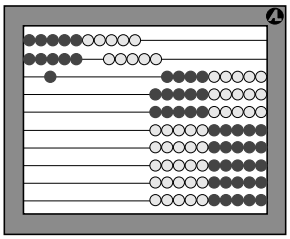


0.1 plus 0.05 = 0.15.

Tell her to add 25 hundredths to the 15 hundredths. See the left figure below. Ask: What are two ways to write the sum using decimals? [0.4 or 0.40]



0.15 + 0.25 = 0.4.



0.15 + 0.06 = 0.21.

Next tell her to clear her abacus then add 15 hundredths and 6 hundreds. [0.21] See the right figure above.

Worksheet 13. Tell the child to complete the worksheet using her abacus. The solutions are below.

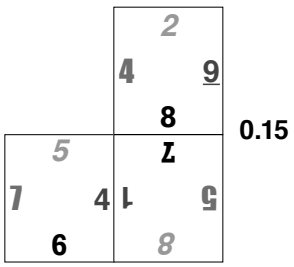
0.2 + 0.15 = **0.35**	**0.07** + 0.4 = **0.47**	**0.7** + 0.04 = **0.74**
0.56 + 0.04 = **0.6**	**0.38** + 0.15 = **0.53**	**0.82** + 0.18 = **1**
0.79 – 0.06 = **0.73**	**0.44** – 0.2 = **0.24**	**1** – 0.37 = **0.63**

Top and Bottom Corners™ with Hundredths game. Play the Top and Bottom Corners™ with Hundredths game, a variation of Top and Bottom Corners™ game, found in *Math Card Games* book, S11. In this game, numbers on the cards are considered to be hundredths. Players take *four* cards to start and take another card after each play.

Record the scores in the math journal. All players start with a score of 5. As usual, players must play to the last card played or to a Corner. They also must play if they can.

In conclusion. Ask: What is the purpose of the decimal point in a number? [It tells where the ones place is.] Which is more, one tenth or ten hundredths? [the same] Which is more, six tenths or sixty hundredths? [the same]

EXPLANATIONS CONTINUED:



Starting with a score of 5 will prevent scores becoming negative.

Name: _____

Date: _____

Warm-Up

Evaluate the following expressions.

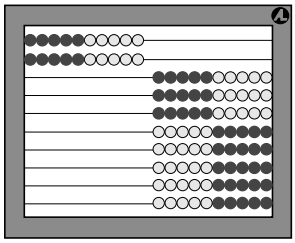
$1^5 + 2^4 + 3^3 =$ _____

$3^3 - 2^4 - 1^5 =$ _____

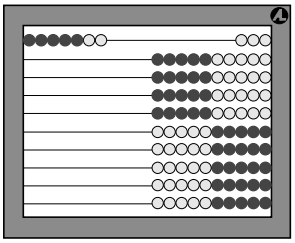
$\frac{3^3}{1^5} - 2^4 =$ _____

$\frac{1^5}{3^3} + 2^4 =$ _____

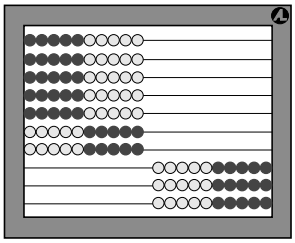
Complete the equations by writing the amount shown on the abacuses in the equation and performing the operations on your abacus. The one hundred beads on the abacus represent 1.



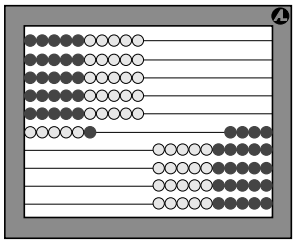
0.2 + 0.15 = _____



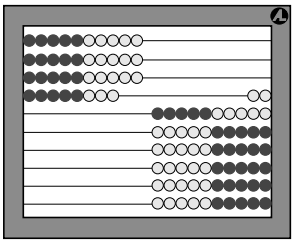
_____ + 0.4 = _____



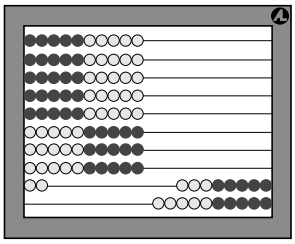
_____ + 0.04 = _____



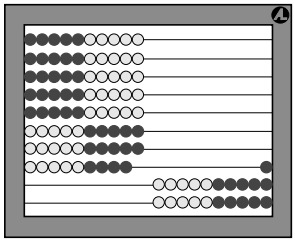
_____ + 0.04 = _____



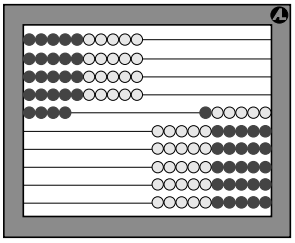
_____ + 0.15 = _____



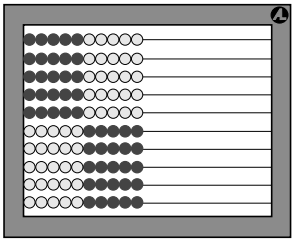
_____ + 0.18 = _____



_____ − 0.06 = _____



_____ − 0.2 = _____



_____ − 0.37 = _____

Lesson 40: Dividing by Tenths

OBJECTIVES:

1. To divide by tenths

MATERIALS:

1. Worksheet 29, Dividing by Tenths
2. AL Abacus

ACTIVITIES FOR TEACHING:

Warm-up. Give the child the worksheet. Tell her to complete just the warm-up problems. Solutions are below.

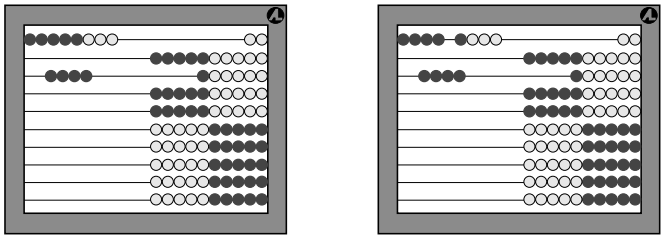
0.85 (4)	38 (2)
× 24 (6)	× 0.49 (4)
340	342
1700	1520
20.40 (6)	18.62 (8)

Dividing whole numbers. Give the child the abacus. Tell her that today she will show division on the abacus in a new way.

Write the expression:

$$\frac{8}{4}$$

and tell her to enter 8 on the top wire. Then tell her to enter 4 on the third wire but to center it under the 8. See the left figure below. Tell her the second wire is similar to the dividing line in the written expression.



With each bead representing 1, the abacuses show eight divided by four, $\frac{8}{4}$.

Ask: How many 4s are in 8? [2] See the second figure above.

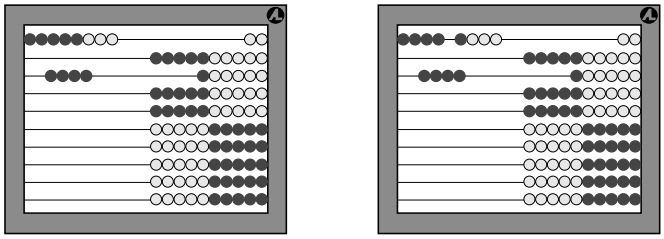
Dividing by tenths. Change the expression to:

$$\frac{.8}{.4}$$

Tell the child to show this expression on her abacus, with each bead standing for one tenth. See the figures at the top of the next page.

EXPLANATIONS:

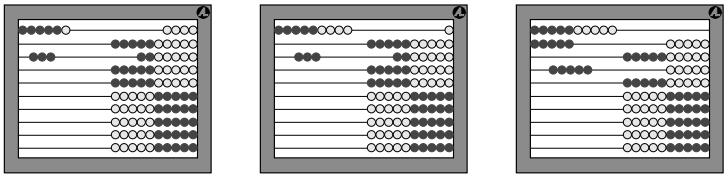
ACTIVITIES FOR TEACHING CONTINUED:



With each bead representing 0.1, the abacuses show eight tenths divided by four tenths, $\frac{.8}{.4}$.

Ask: How many four tenths are in eight tenths? [2] See the right figure above.

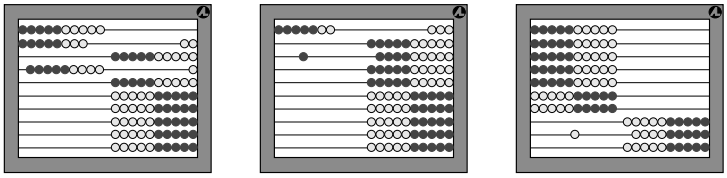
Worksheet 29. Tell the child to complete the first row on the worksheet. The solutions are shown below.



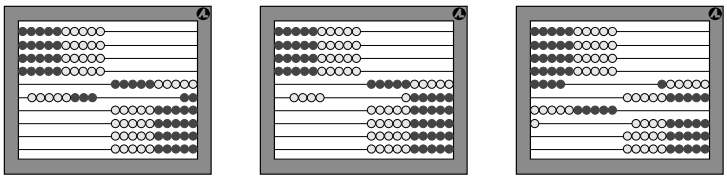
$\frac{.6}{.3} = 2$ $\frac{.9}{.3} = 3$ $\frac{1.5}{0.5} = 3$

Ask her to explain how she found her answer. In the third example, the first row is 1 because ten tenths equal 1.

Then tell her to complete the worksheet. The solutions are below.



$\frac{1.8}{.9} = 2$ $\frac{.7}{.1} = 7$ $\frac{7}{0.1} = 70$



$\frac{4.0}{.8} = 5$ $\frac{4.0}{.4} = 10$ $\frac{4.4}{1.1} = 4$

If each bead in the abacuses above suddenly explodes becoming ten times greater, what happens to your answers?

They stay the same.

In conclusion. Ask: Is 8 tenths divided by 2 tenths the same as 8 divided by 2? [yes] Is 9 tenths divided by 3 tenths the same as 9 divided by 3? [yes] Is 8 hundredths divided by 2 hundredths the same as 8 divided by 2? [yes]

EXPLANATIONS CONTINUED:

If there is additional time following this lesson, play the Subtraction Corners™ with Tenths game, found in *Math Card Games* book, F22.4.

Name: _____

Date: _____

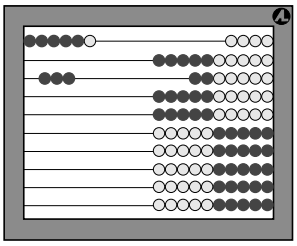
Warm-Up

Multiply the numbers given. Then divide by the same numbers. Use check numbers to check your work if you like.

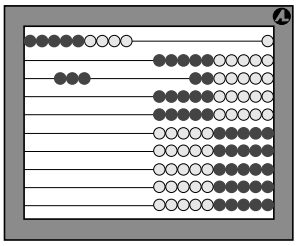
		0	.8	5	( )
		×	2	4	( )
					( )

			3	8	( )
	×	0	.4	9	( )
					( )

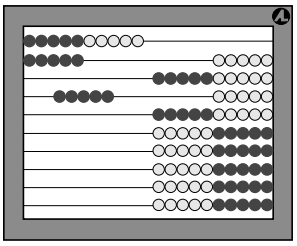
Write the equations shown on the abacuses. Each bead on the abacus represents 0.1.



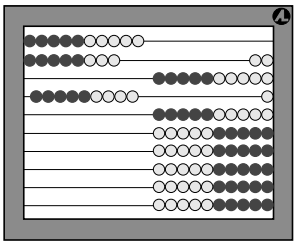
$\frac{0.6}{0.3}$ = _____



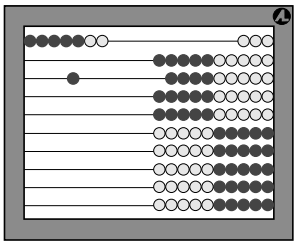
_____ = _____



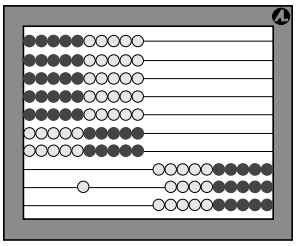
_____ = _____



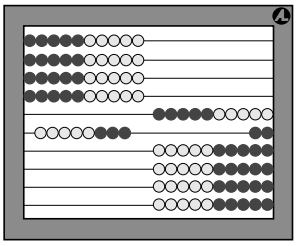
_____ = _____



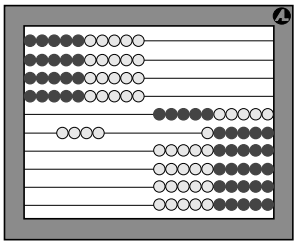
_____ = _____



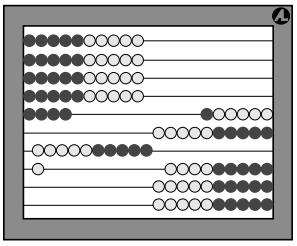
_____ = _____



_____ = _____



_____ = _____



_____ = _____

If each bead in the abacuses above suddenly explodes becoming ten times greater, what happens to your answers? _____

Lesson 51: Remainder Forms after Dividing

OBJECTIVES:

1. To divide by subtracting
2. To write remainders in three forms: whole number, fraction, and decimal

MATERIALS:

1. Worksheet 39, Remainder Forms after Dividing
2. Casio SL-450S calculator

ACTIVITIES FOR TEACHING:

Warm-up. Give the child the worksheet. Tell her to do just the warm-up problems. Solutions are below.

```
     (7) (5)          (4) (2)
     340 r23           85 r29
28)9543 (3)      73)6234  (6)
(1)              (1)
```

Dividing by subtracting. Give the child the calculator. Tell her that today she will do some division with a calculator.

Write: $160 \div 32 =$

and tell her she is to use her calculator, but she cannot use the division key. Give her a few minutes to work on the problem independently. Then tell her to discuss the problem.

There are several solutions. One way is to start with 160 and subtract 32s until reaching zero; each subtraction needs to be counted. The constant feature makes it easier. Start by pressing 32, then [−] [−]. Next enter 160 and press [=], which subtracts 32. Continue pressing [=] until the remainder is less than 32, in this case, 0.

Another way is to start with 32 and add 32s until reaching 160, which will be a total of 5 times. Using the constant feature will make this easier, too.

Dividing by a two-digit number using subtraction. Now tell the child to complete the following problem with her calculator, but without using the division key.

$$864 \div 32 =$$

Give her time to solve it before discussing the solution.

While it is possible to subtract 32 twenty-seven times, it is simpler to subtract 320 twice (32 × 10), and then subtract 32 seven times.

Worksheet 39. Tell the child to complete the first problem on the worksheet. Solutions are shown on the next page.

EXPLANATIONS:

Some children may need a hint that they could subtract groups of ten 32s.

ACTIVITIES FOR TEACHING CONTINUED:

414 ÷ 18 = **23**

414 – (18 × 10) – (18 × 10) – 18 – 18– 18

1728 ÷ 54 = **32**

1728 – (54 × 10) – (54 × 10) – (54 × 10) – 54 – 54

Remainders in other forms. Tell her to read and solve Problem 2.

2. In a leap year, 366 days, what is the average number of days in a month? Give the answer in days and a fraction of a day.

Then tell her to discuss the solution.

$$12\overline{)366}\quad 30\ \tfrac{6}{12}$$

Ask: Is there a simpler fraction that is equal $\frac{6}{12}$? [$\frac{1}{2}$] Now tell her to use her calculator to find 366 divided by 12, using the division key. [30.5] Ask: Did you get the same answer? [yes, since $\frac{1}{2}$ is the same as .5]

Tell her to complete Problem 3.

3. In a non-leap year, what is the average number of days in a month? Give the answer in days and a fraction of a day.

$$12\overline{)365}\quad 30\ \tfrac{5}{12}$$

Now tell her to use her calculator to find 365 divided by 12. [30.416666] Ask: Did you get the same answer? [yes, since $\frac{5}{12}$ is the same as .4166666]

Worksheet. Tell the child to complete the worksheet. The solutions are below.

	Remainder as a Whole Number	Remainder as a Fraction	Remainder as a Decimal
7 ÷ 3	2 r1	$2\frac{1}{3}$	2.33
13 ÷ 8	**1 r5**	**$1\frac{5}{8}$**	**1.63**
51 ÷ 7	**7 r2**	**$7\frac{2}{7}$**	**7.29**
1001 ÷ 25	**40 r1**	**$40\frac{1}{25}$**	**40.04**
6983 ÷ 86	**81 r17**	**$81\frac{17}{86}$**	**81.20**
3078 ÷ 12	**256 r6**	**$256\frac{6}{12}$**	**256.50**

In conclusion. Ask: Is it possible to do division without any multiplying? [yes] What operation would you use? [subtraction] Is that the easiest way? [no] What are the three forms for writing remainders? [whole number, fraction, and decimal]

EXPLANATIONS CONTINUED:

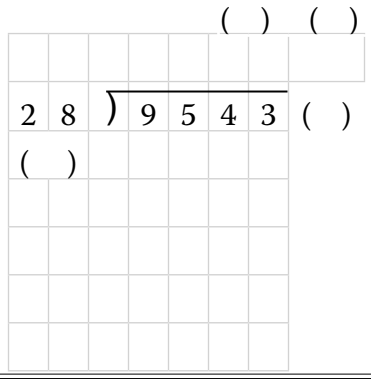
Dividing $\frac{5}{12}$ on a calculator gives 0.4166666. Some children may need to divide 12 by 5 on the calculator to see that $\frac{5}{12}$ is 0.4166666.

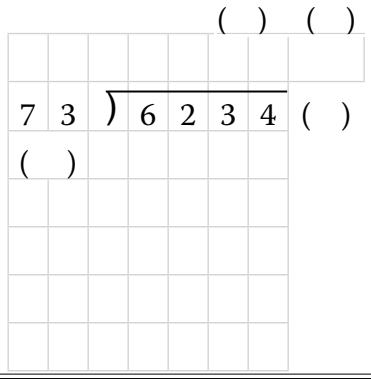
Name: _____

Date: _____

Warm-Up

Divide and check your answers with check numbers.





1. Use your calculator, but not the ÷ key, to find the following. Explain what you did.

414 ÷ 18 _____

_____

1728 ÷ 54 _____

_____

2. In a leap year, 366 days, what is the average number of days in a month? Give the answer in days and a fraction of a day.

3. In a non-leap year, what is the average number of days in a month? Give the answer in days and a fraction of a day.

4. Divide using your calculator as needed. Write the quotients with remainders three different ways. Round the decimals to hundredths.

	Remainder as a Whole Number	Remainder as a Fraction	Remainder as a Decimal
7 ÷ 3	2 rl	$2\frac{1}{3}$	2.333
13 ÷ 8			
51 ÷ 7			
1001 ÷ 25			
6983 ÷ 86			
3078 ÷ 12			

Lesson 62: Area of Parallelograms

OBJECTIVES:

1. To find the area of parallelograms

MATERIALS:

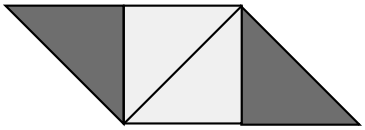
1. Warm-up Practice 4
2. Worksheet 50, Area of Parallelograms
3. A sets of tangrams
4. 45 triangle (or 30-60 triangle), optional

ACTIVITIES FOR TEACHING:

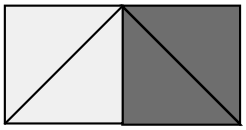
Warm-up. Give the child the warm-up practice sheet. Tell her to complete the second multivide and long division problem. Solutions are on the right.

Area of a parallelogram. Give the child the worksheet and tangrams. Tell her that today's lesson is about finding areas of parallelograms.

Tell her to make a parallelogram with four tangram triangles of the same size, either the large triangles or the small triangles. See the left figure below.



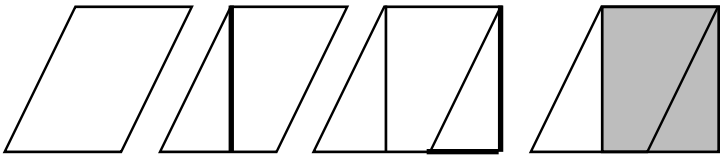
A parallelogram.



Rectangle with same area.

Now tell her to move one piece and to turn it into a rectangle. See the right figure above. Ask: Does the rectangle have the same area as your original parallelogram? [yes] How do you know? [They are the same pieces, just moved.] How can you find the area of the rectangle? [multiply the width times the height] Could we use the width times the height to find the area of the parallelogram? [yes]

Draw a parallelogram as shown below in the left figure.



Say: Let's find the area of the parallelogram. First turn it into a rectangle having the same area. Draw a line from the top left corner to the base as shown in the second figure above. Ask: To where do we need to move the triangle? [to the right side] See the third figure. Shade in the rectangle as shown in the fourth figure. Ask: Does this rectangle have the same area as the parallelogram? [yes]

EXPLANATIONS:

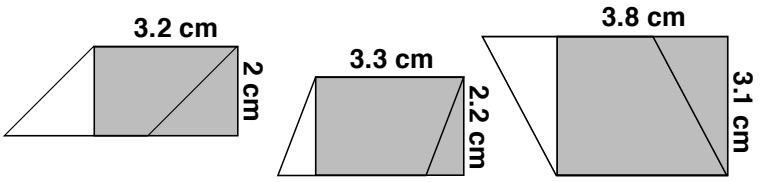
A parallelogram is a quadrilateral with opposite sides parallel.

```
      468  (0)
  × 0.24   (6)
     1872
     9360
   112.32  (0)
  × 0.72   (0)
    22464
   786240
0.6)80.8704  (0)
0.4)134.784  (0)
0.8)336.96   (0)
0.9)421.2    (0)
      468

        (3)  (8)
         30  r8
82)2468  (2)
  (1)  246
        08
         0
         8
```

ACTIVITIES FOR TEACHING CONTINUED:

Worksheet 50, Problem 1. Tell the child to solve the first row on the worksheet. The solutions are below.



$A = 3.2 \times 2$
$A = 6.4 \text{ cm}^2$

$A = 3.3 \times 2.2$
$A = 7.3 \text{ cm}^2$

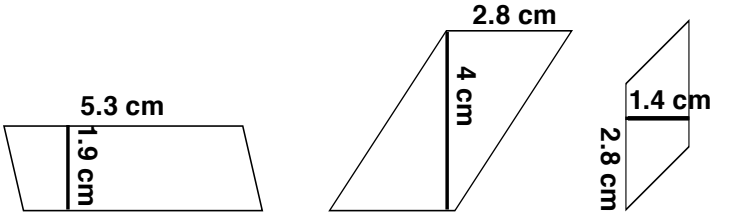
$A = 3.8 \times 3.1$
$A = 11.8 \text{ cm}^2$

Tell her to explain why the area of a parallelogram can be found by multiplying the width by the height. [The area of the rectangle has the same area as the parallelogram.]

Finding the heights in a parallelogram. Draw a parallelogram as shown in the left figure and ask: How can you find the height of this parallelogram without drawing a rectangle? [Draw a line perpendicular to the width.] Draw several heights as shown in the right figure and ask: Which height should we use? [any of them]

A parallelogram **A parallelogram with heights.**

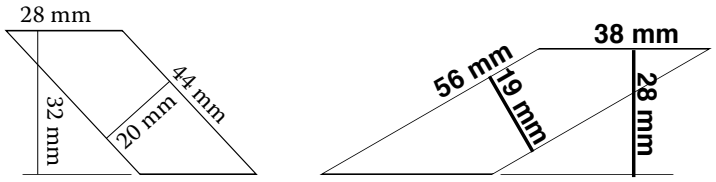
Problems 2 and 3. Tell the child to complete the worksheet. The solutions are below.



$A = 5.3 \times 1.9$
$A = 10.1 \text{ cm}^2$

$A = 2.8 \times 4$
$A = 11.2 \text{ cm}^2$

$A = 1.4 \times 2.8$
$A = 3.9 \text{ cm}^2$



$A = 28 \times 32 = 896 \text{ mm}^2$
$A = 44 \times 20 = 880 \text{ mm}^2$

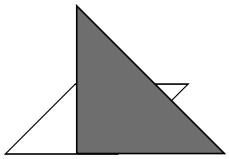
$A = 38 \times 28 = 1064 \text{ mm}^2$
$A = 56 \times 19 = 1064 \text{ mm}^2$

Ask: Why do you think the two areas for the first figure in Problem 3 are not identical? [Measurements are not exact.]

In conclusion. Ask: How do you find the area of a parallelogram? [multiply width times height] What is the height? [line that is perpendicular to the width]

EXPLANATIONS CONTINUED:

To be more accurate in drawing the perpendicular lines, the child could use either a tangram triangle or a triangle from the drawing set. See below.



Measurements given here are accurate, but worksheet measurements may vary and will affect the final answers.

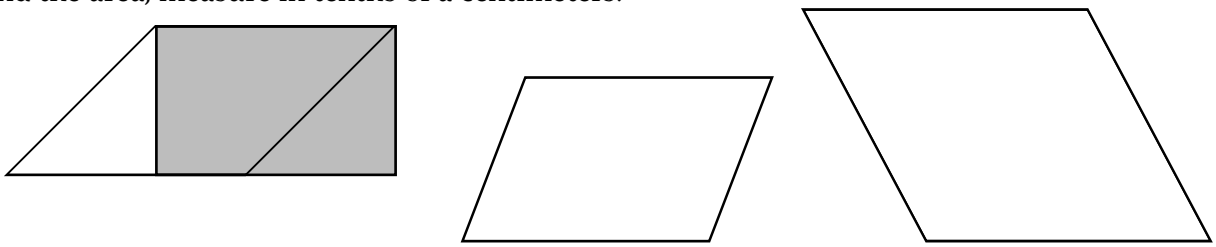
The fact that the area calculations for the same figure do not always give the identical answer is often surprising and sometimes upsetting to some people.

If there is additional time following this lesson, play the Old Main Squares game, found in *Math Card Games* book, P22.

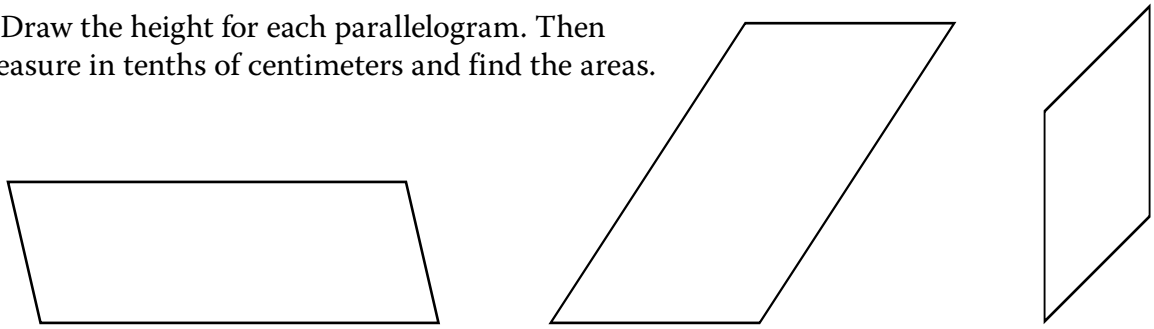
Name: _____

Date: _____

1. Turn the following parallelograms into rectangles. Crosshatch or shade the rectangle. Then find the area; measure in tenths of a centimeters.



2. Draw the height for each parallelogram. Then measure in tenths of centimeters and find the areas.



3. Find the area of the parallelograms in two different ways. Use millimeters.